Implicit positivity-preserving high order discontinuous Galerkin methods for conservation laws

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Positivity-preserving discontinuous Galerkin (DG) methods for solving hyperbolic conservation laws have been extensively studied in the last several years. But nearly all the developed schemes are coupled with explicit time discretizations. Explicit discretizations suffer from the constraint for the Courant-Friedrichs-Levis (CFL) number. This makes explicit methods impractical for problems involving unstructured and extremely varying meshes or long-time simulations. Instead, implicit DG schemes are often popular in practice, especially in the computational fluid dynamics (CFD) community. In this talk we discuss a high-order positivity-preserving DG method with the backward Euler time discretization for conservation laws. Both the analysis for the simple linear case and numerical experiments for nonlinear systems indicate that a lower bound for the CFL number is required to obtain the positivity-preserving property. The proposed scheme not only preserves the positivity of the numerical approximation without compromising the designed high-order accuracy, but also helps accelerate the convergence towards the steady-state solution and add robustness to the nonlinear solver. This is a joint work with Tong Qin.