MIT AEROASTRO

DATA to DECISIONS

computational methods for the **next generation** of **aerospace systems**

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FUTURE CFD TECHNOLOGIES WORKSHOP

Bridging Mathematics and Computer Science for Advanced Aerospace Simulation Tools

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MIT AEROASTRO

MIT | the center for computational engineering

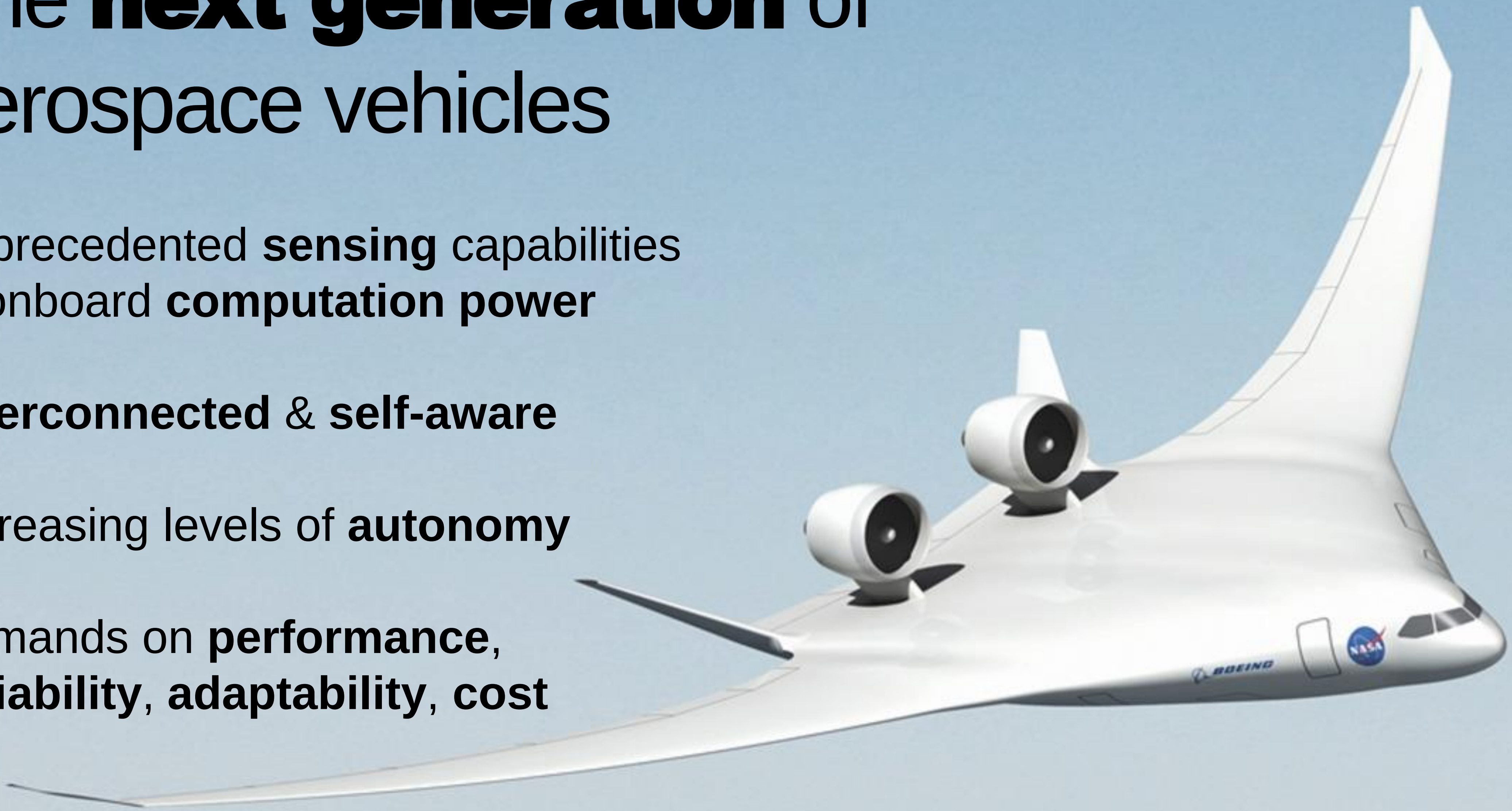
The **next generation** of aerospace vehicles

unprecedented **sensing** capabilities
& onboard **computation power**

interconnected & **self-aware**

increasing levels of **autonomy**

demands on **performance**,
reliability, **adaptability**, **cost**

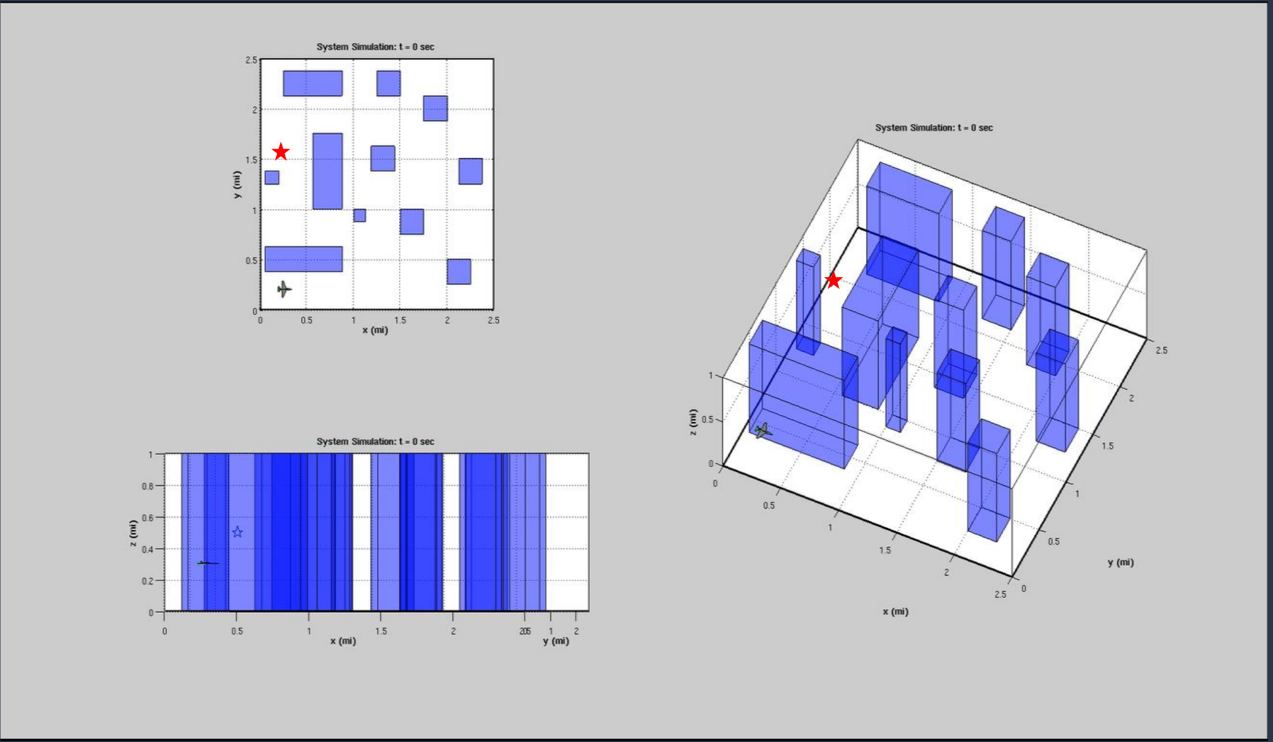


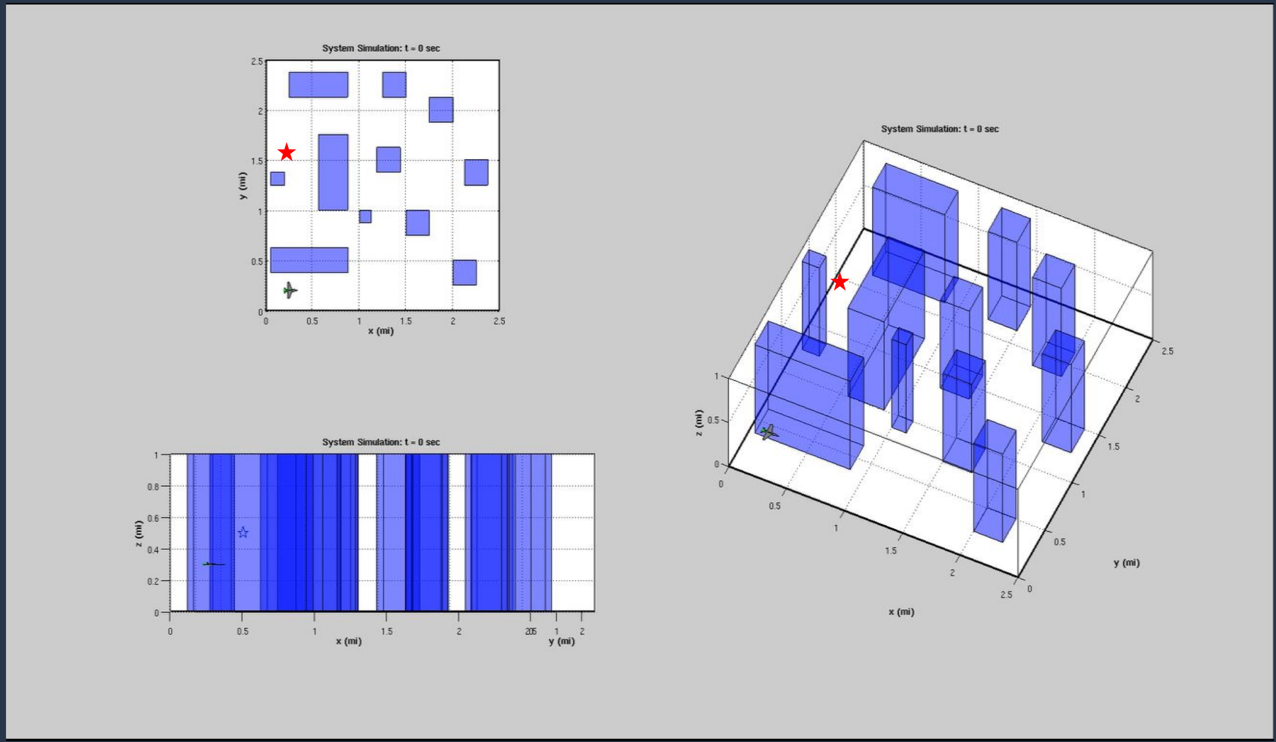
new **technologies** + **data**
+ **computation** power

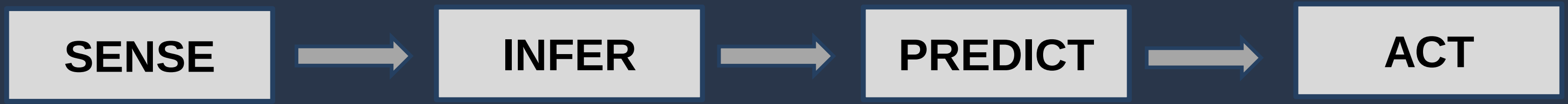
=

new ways to think about design
for **efficiency**, **cost**, **reliability, …**

Data + Models:
self-aware aerospace vehicles





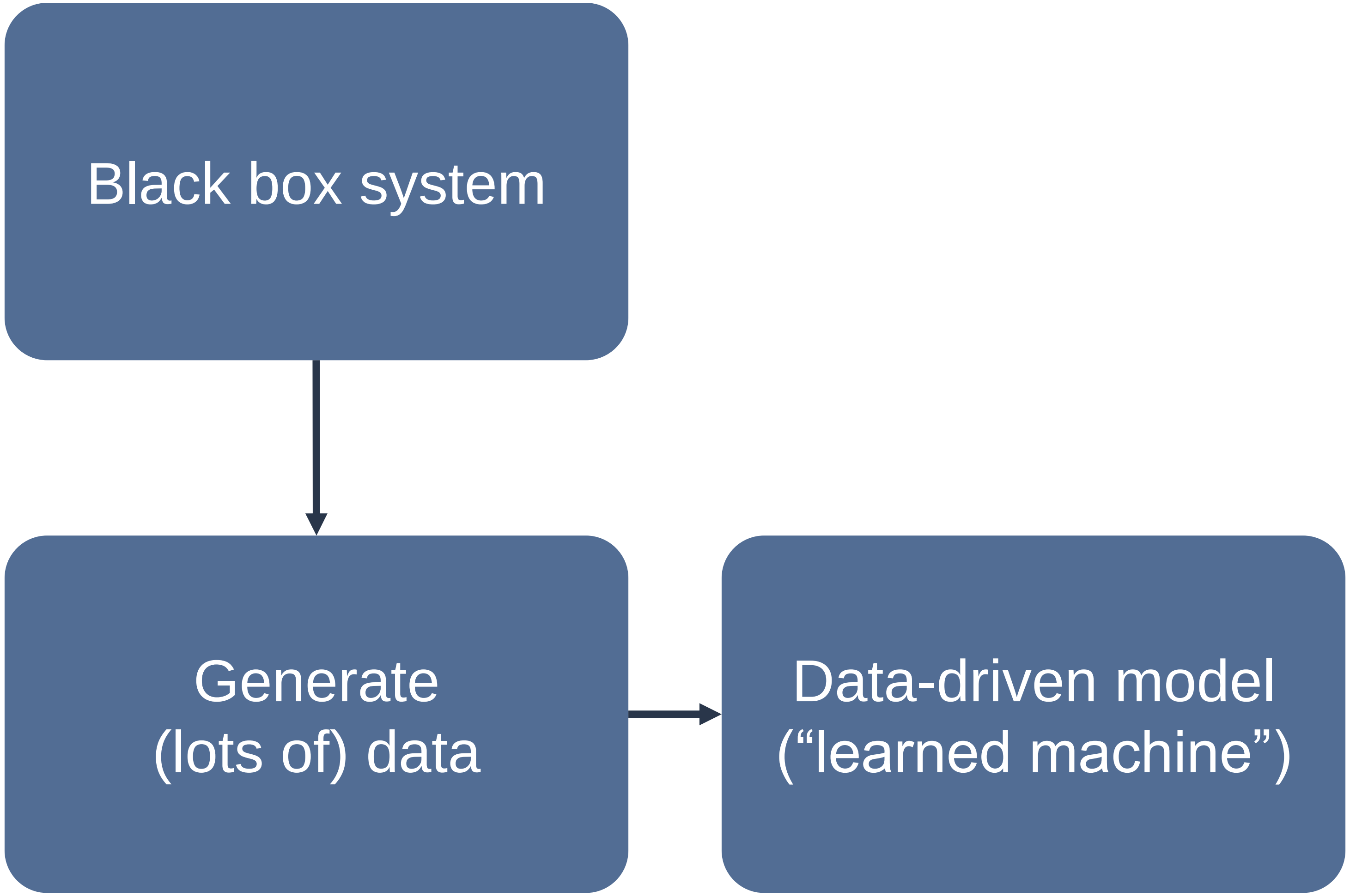


Singh & W., *AIAA J.*, 2017

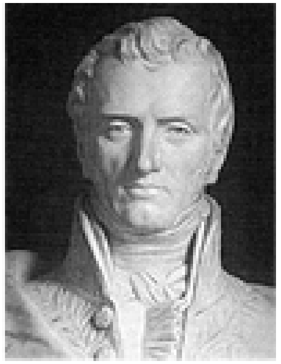
Developing **mathematical foundations** and **computational methods** to enable design of the next generation of engineered systems.

1 modeling the data-to-decisions flow **2** exploiting synergies between physics-based models & data **3** principled approximations to reduce computational cost **4** explicit modeling & treatment of uncertainty

Machine learning: A powerful toolbox for modeling



Machine learning: A powerful toolbox but can we trust the models?

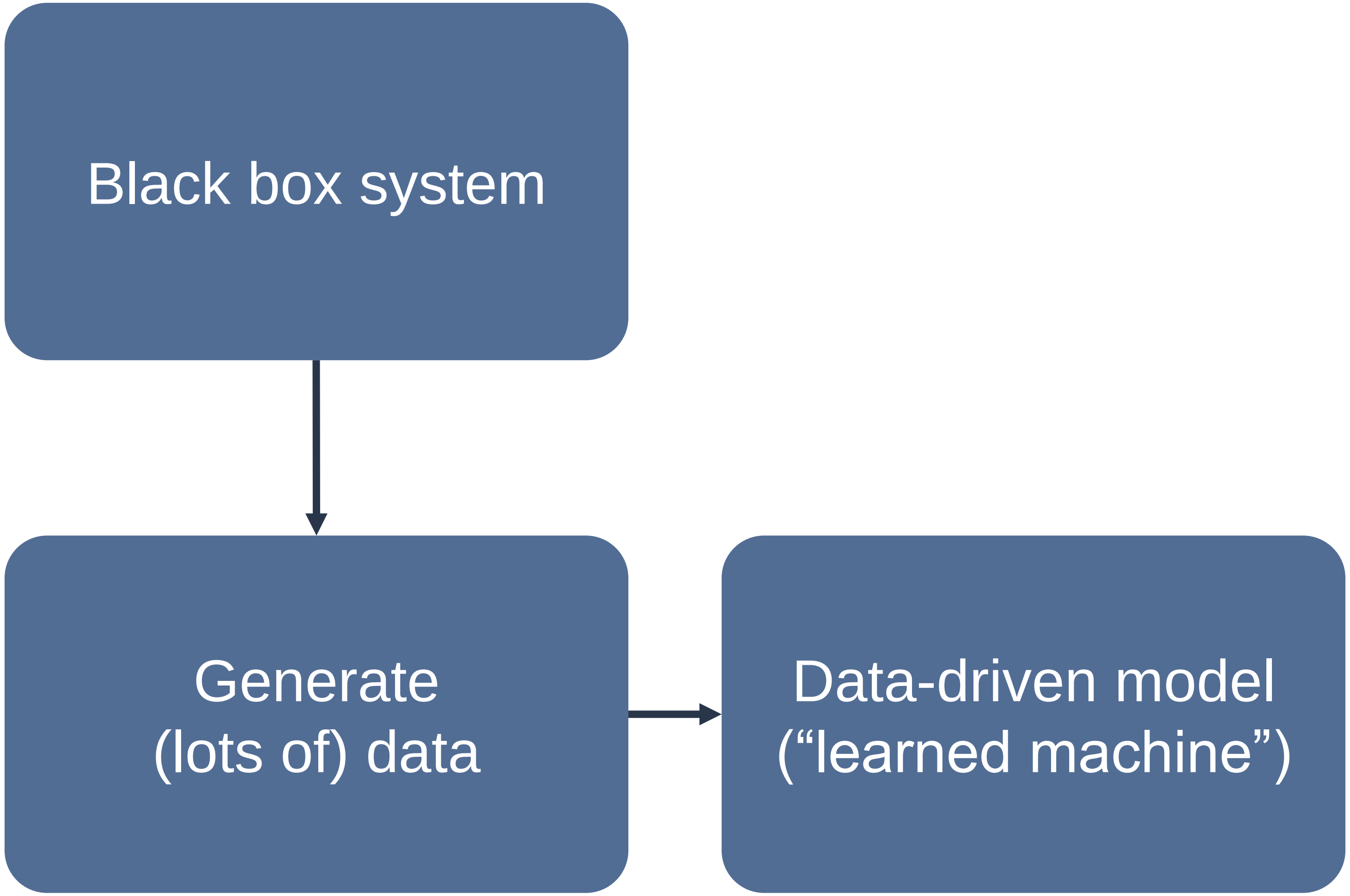


Claude-Louis Navier

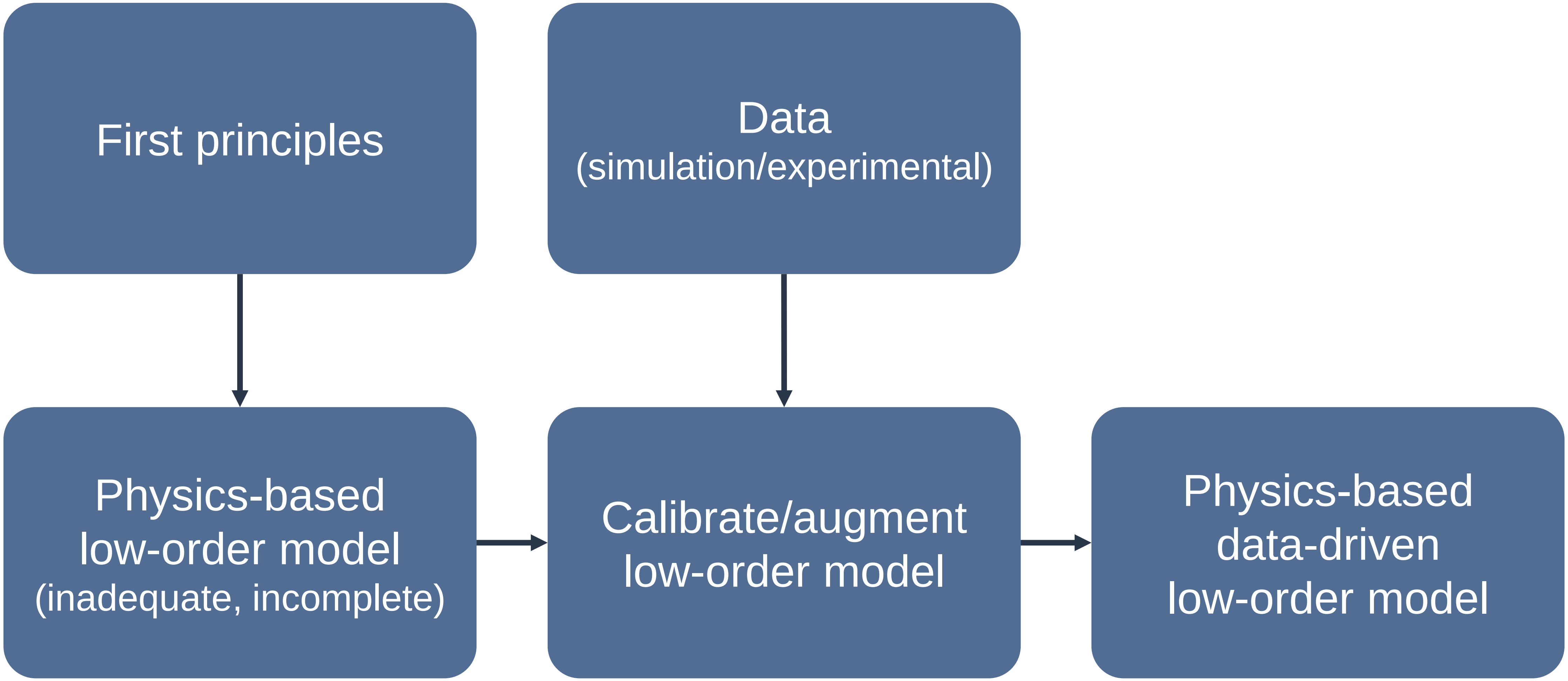
Sir George Gabriel Stokes

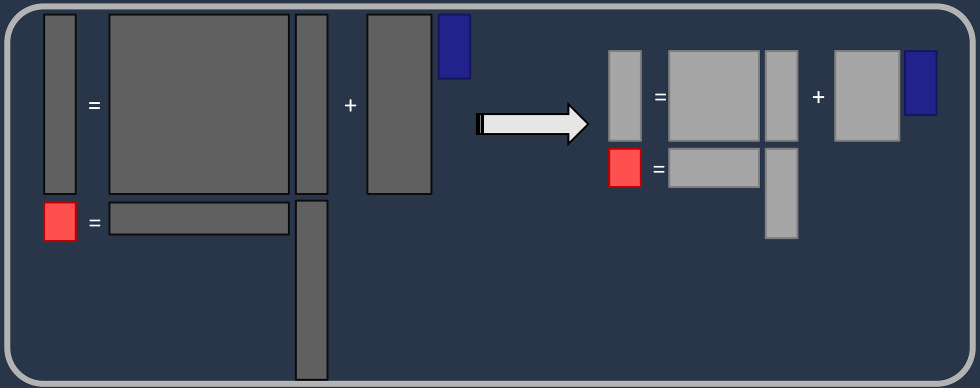
Leonhard Euler
(1707-1783)

Sir Isaac Newton
(1642-1727)



Our approach: Leverage physics-based reasoning + data-driven learning





Projection-based Model Reduction

physics-based mathematical framework to extract the essence of complex problems

Start with a physics-based model

large-scale and expensive to solve

Arising, for example, from systems of ODEs or spatial discretization of PDEs describing the system of interest

- which in turn arise from governing physical principles (conservation laws, etc.)

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}(\mathbf{p})\mathbf{u} \\ \mathbf{y} &= \mathbf{C}(\mathbf{p})\mathbf{x} \end{aligned}$$

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{p}, \mathbf{u}) \\ \mathbf{y} &= g(\mathbf{x}, \mathbf{p}, \mathbf{u}) \end{aligned}$$

$\mathbf{x} \in \mathbf{R}^{N}$: state vector

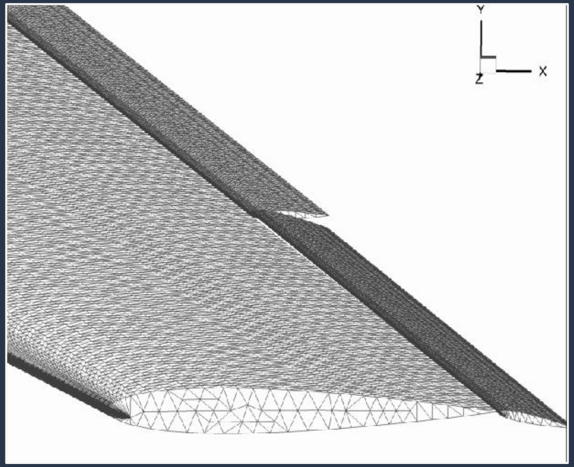
$\mathbf{u} \in \mathbf{R}^{N_i}$: input vector

$\mathbf{p} \in \mathbf{R}^{N_p}$: parameter vector

$\mathbf{y} \in \mathbf{R}^{N_o}$: output vector

Example: CFD systems

modeling the flow over an aircraft wing



$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}(\mathbf{p})\mathbf{u} \\ \mathbf{y} &= \mathbf{C}(\mathbf{p})\mathbf{x} \end{aligned}$$

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{p}, \mathbf{u}) \\ \mathbf{y} &= g(\mathbf{x}, \mathbf{p}, \mathbf{u}) \end{aligned}$$

- $\mathbf{x}(t)$: vector of N flow unknowns
  e.g., 2D incompressible Navier Stokes
  P grid points, $N = 3P$
  $\mathbf{x} = [u_1\ v_1\ p_1\ u_2\ v_2\ p_2 \cdots\cdots u_P\ v_P\ p_P]^T$
- $\mathbf{p}$: input parameters
  e.g., shape parameters, PDE coefficients
- $\mathbf{u}(t)$: forcing inputs
  e.g., flow disturbances, wing motion
- $\mathbf{y}(t)$: outputs
  e.g., flow characteristic, lift force

Reduced models

low-cost but accurate approximations of high-fidelity models via projection onto a low-dimensional subspace

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}(\mathbf{p})\mathbf{u} \\ \mathbf{y} &= \mathbf{C}(\mathbf{p})\mathbf{x} \end{aligned}$$

$\mathbf{x} \approx \mathbf{V}\mathbf{x}_r$

$$\begin{aligned} \mathbf{r} &= \mathbf{V}\dot{\mathbf{x}}_r - \mathbf{A}\mathbf{V}\mathbf{x}_r - \mathbf{B}\mathbf{u} \\ \mathbf{y}_r &= \mathbf{C}\mathbf{V}\mathbf{x}_r \end{aligned}$$

$\mathbf{W}^T\mathbf{r} = 0$

$$\begin{aligned} \dot{\mathbf{x}}_r &= \mathbf{A}_r(\mathbf{p})\mathbf{x}_r + \mathbf{B}_r(\mathbf{p})\mathbf{u} \\ \mathbf{y}_r &= \mathbf{C}_r(\mathbf{p})\mathbf{x}_r \end{aligned}$$

$$\begin{aligned} \mathbf{A}_r(\mathbf{p}) &= \mathbf{W}^T\mathbf{A}(\mathbf{p})\mathbf{V} \\ \mathbf{B}_r(\mathbf{p}) &= \mathbf{W}^T\mathbf{B}(\mathbf{p}) \\ \mathbf{C}_r(\mathbf{p}) &= \mathbf{C}(\mathbf{p})\mathbf{V} \end{aligned}$$

$\mathbf{x} \in \mathbf{R}^N$: state vector

$\mathbf{p} \in \mathbf{R}^{N_p}$: parameter vector

$\mathbf{u} \in \mathbf{R}^{N_i}$: input vector

$\mathbf{y} \in \mathbf{R}^{N_o}$: output vector

$\mathbf{x}_r \in \mathbf{R}^n$: reduced state vector

$\mathbf{V} \in \mathbf{R}^{N\times n}$: reduced basis

Classically

- Reduced models are built and used in a **static** way:
  - offline phase: sample a high-fidelity model, build a low-dimensional basis, project to build the reduced model
  - online phase: use the reduced model

Data-driven reduced models

- Recognize that conditions may change and/or initial reduced model may be inadequate
  - offline phase: build an initial reduced model
  - online phase: **learn** and **adapt** using dynamic data

Data-driven reduced models

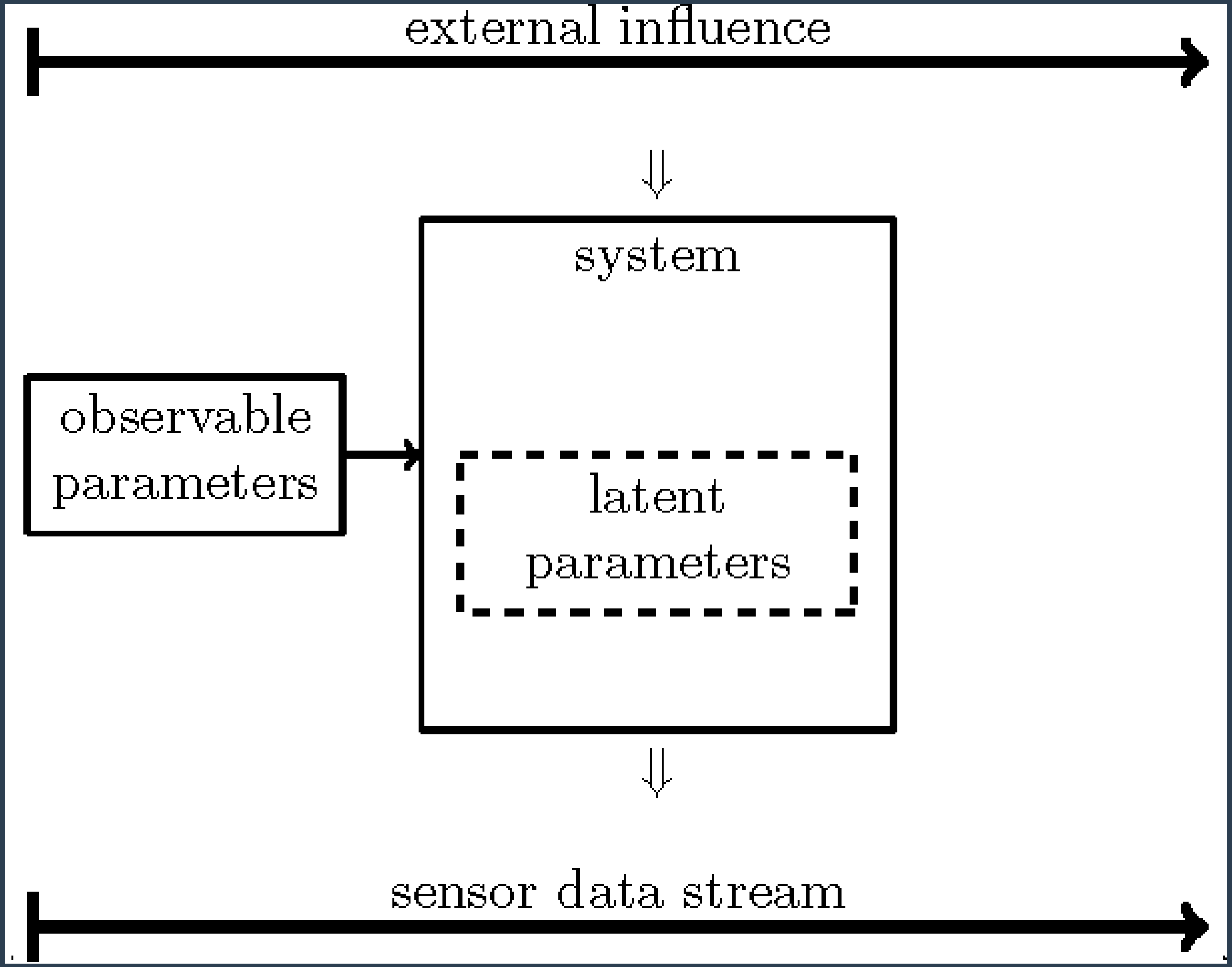
exploiting the synergies of physics-based models and dynamic data

- **Adaptation** and **learning** are **data-driven**
  - sensor data collected online
    (e.g., structural sensors on board an aircraft)
  - simulation data collected online
    (e.g., over the path to an optimal solution)

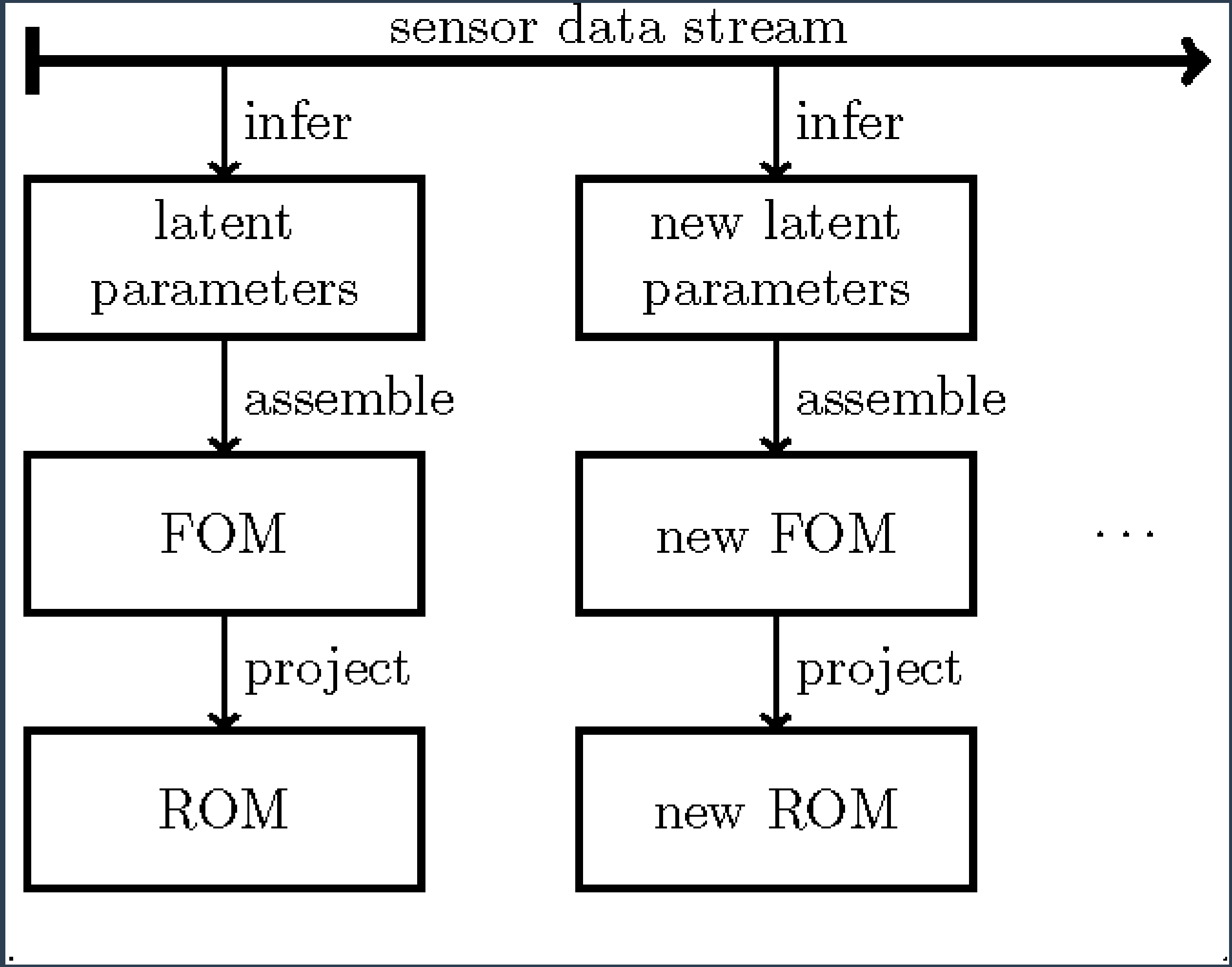
but the **physics-based model** remains as an underpinning.

- Achieve **adaptation** in a variety of ways:
  - adapt the basis *(Cui, Marzouk, W., 2014)*
  - adapt the way in which nonlinear terms are approximated *(Peherstorfer, W., 2015)*
  - adapt the reduced model itself *(Peherstorfer, W., 2015)*
  - construct localized reduced models; adapt model choice *(Peherstorfer, Butnaru, W., Bungartz, 2014; Mainini, W., 2015)*
  - directly infer the reduced model *(Peherstorfer, W., 2016)*

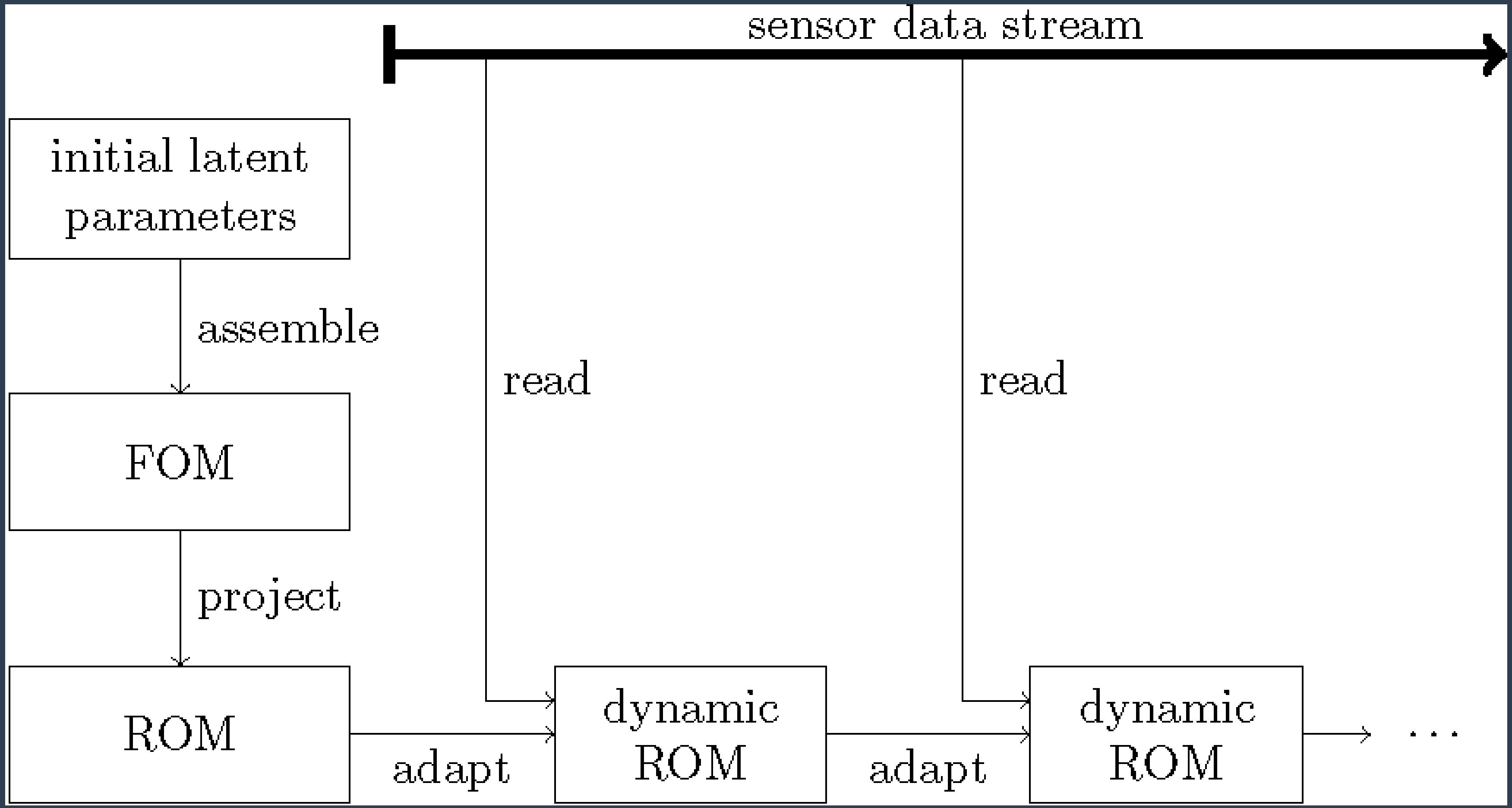
Consider a system with **observable** and **latent parameters**



Classical approaches build the new reduced model from scratch

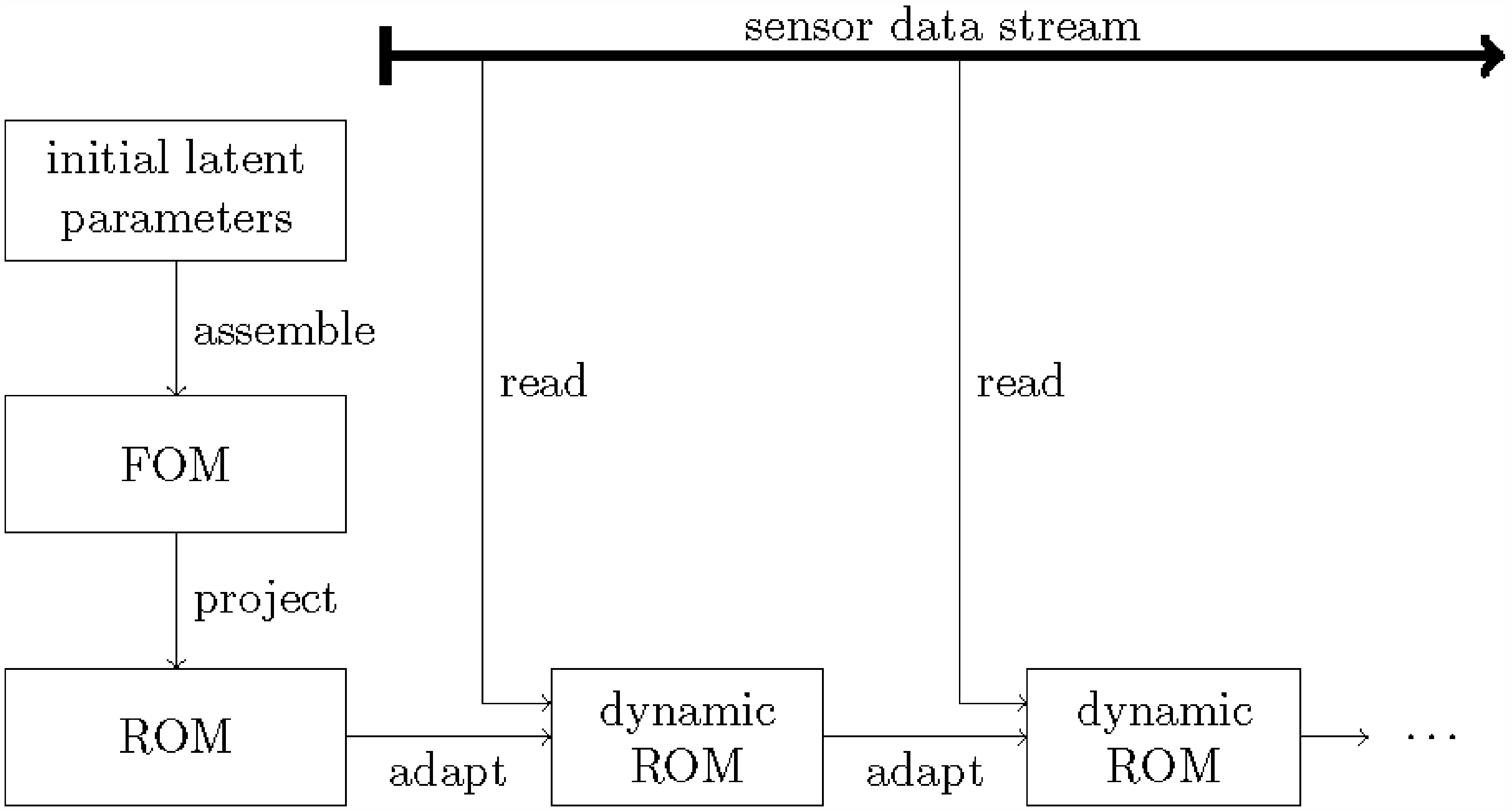


A **dynamic reduced model** adapts in response to the data, without recourse to the full model



Data-driven reduced models

- **adapt** directly from sensor data
- **avoid** (expensive) inference of latent parameter
- **avoid** recourse to full model

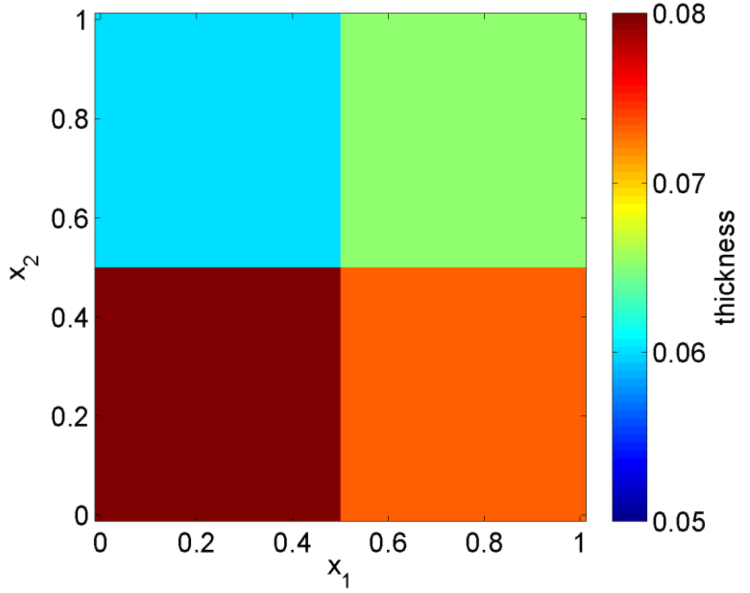


- incremental SVD methods (exploit structure of a rank-one snapshot update)
- convergence guarantees in idealized noise-free case

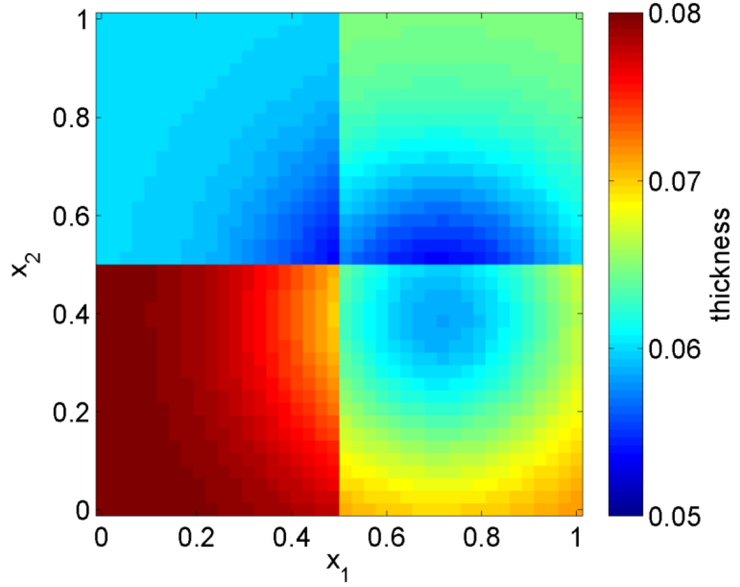
Example: locally damaged plate

High-fidelity:
finite element model

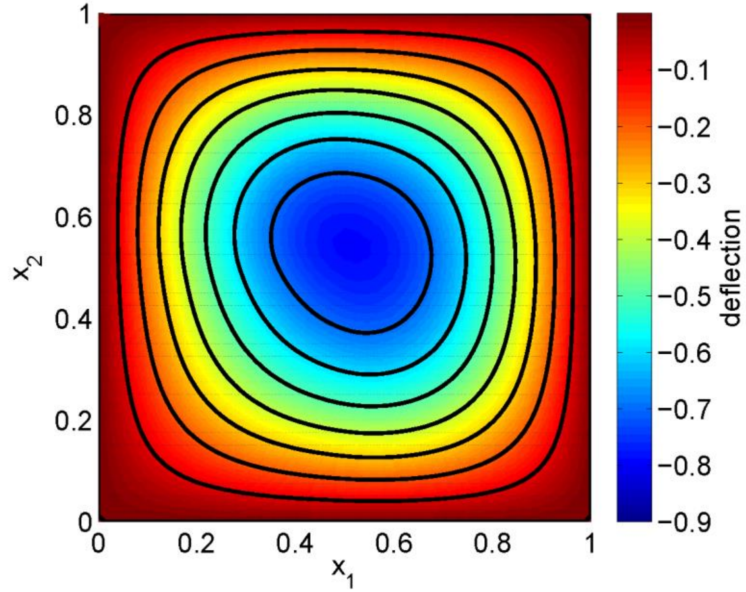
Reduced model:
proper orthogonal decomposition



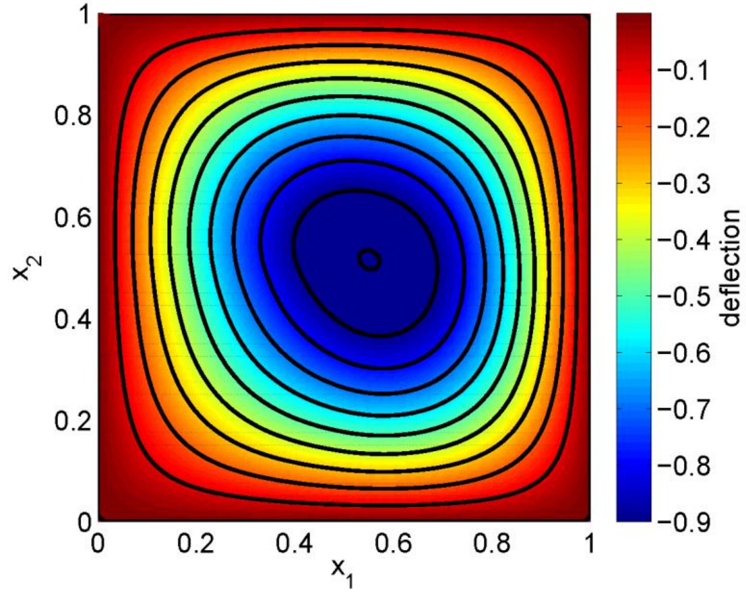
thickness, no damage



thickness, damage up to 20%



deflection, no damage

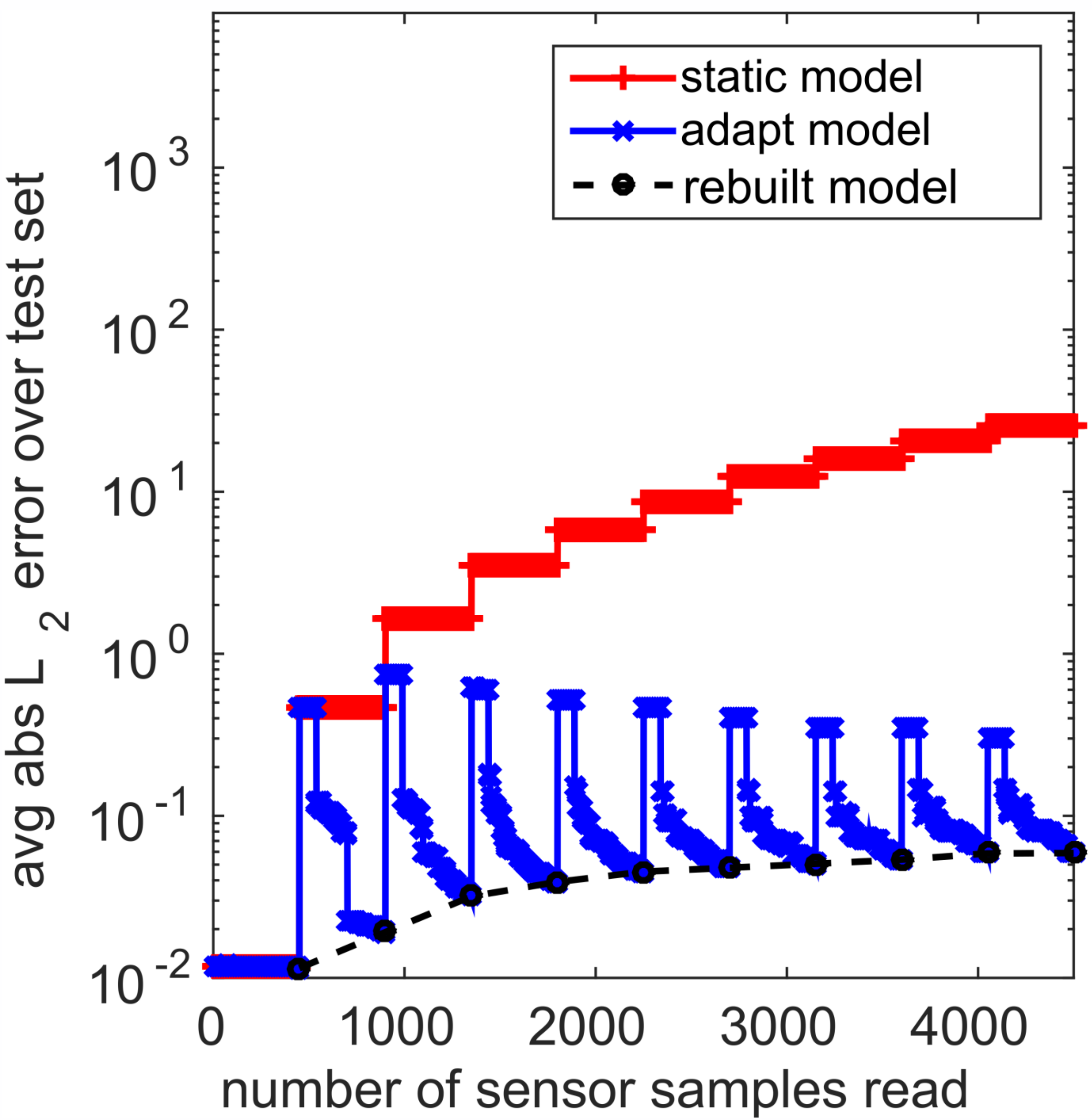


deflection, damage up to 20%

Data-driven adaptation: locally damaged plate

Adapting the ROM after damage

Speedup of 10^4 cf. rebuilding ROM



Multifidelity models and multifidelity methods

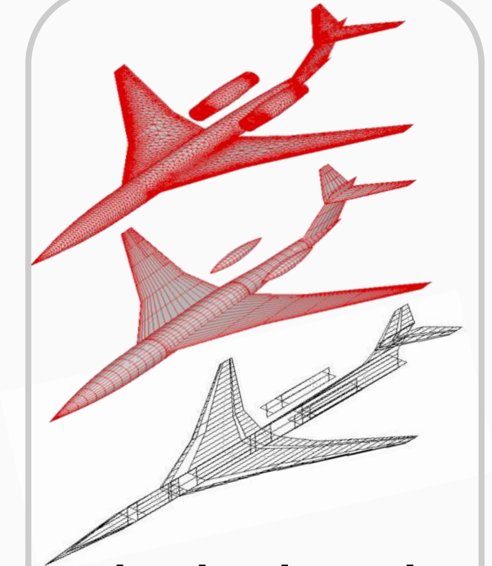
“All models are wrong, but some are useful.”

George Box, 1979

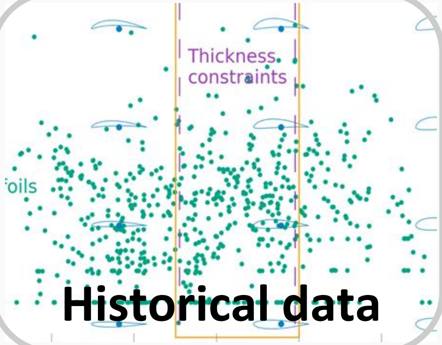
Multifidelity models

analysis and design typically **begin with low-fidelity** models and progressively incorporate higher fidelity tools

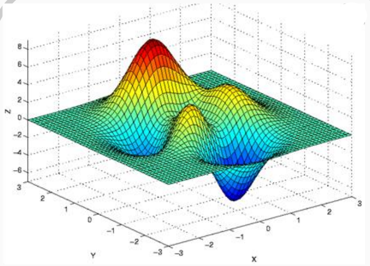
Many information sources available: multi-fidelity models, historical data, operational data, experimental data, expert opinions



Physics-based models



Historical data



Surrogate & reduced models

Source: wikipedia.org

Experts

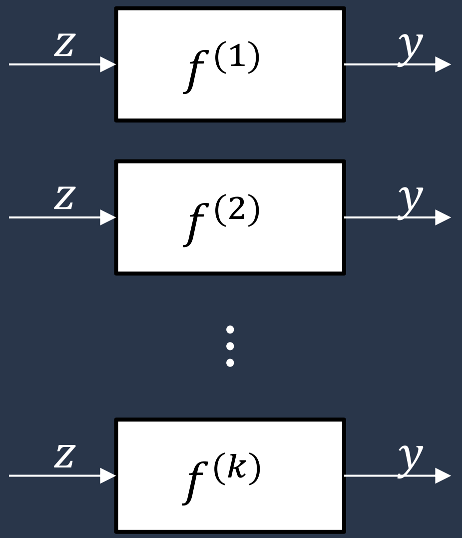
Experimental data

Telling us different things about the system:
the collective information is greater than the individual parts

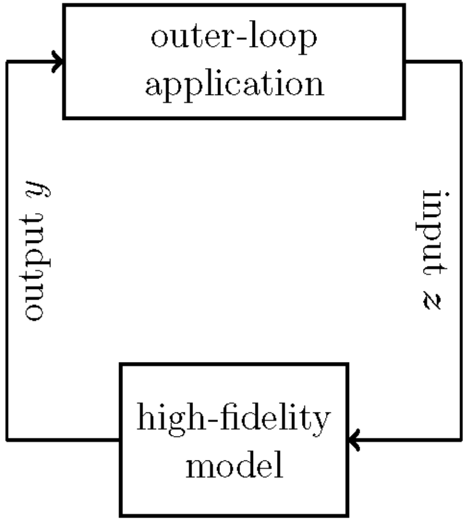
Critical to get the right information early in the decision process

Multifidelity methods for outer-loop problems

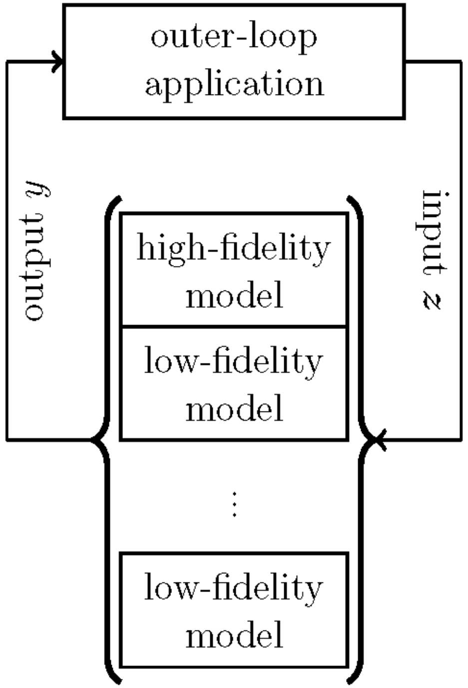
Peherstorfer, W., Gunzburger, *SIAM Review (*Survey), 2018



- **Outer-loop**: computational applications that form outer loops around a model
  – overall outer-loop result is obtained at the termination of the outer loop
  – examples: optimization, uncertainty propagation, inverse problems, data assimilation, control, sensitivity analysis

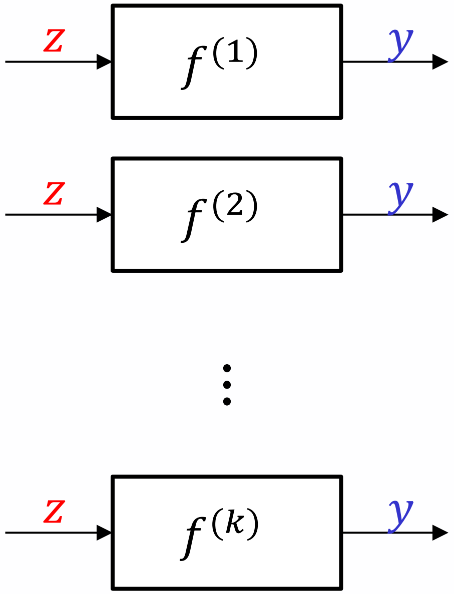


- **Multifidelity methods**: goal is to solve the outer-loop problem at high fidelity
  – invoke multiple models to reduce computational cost
  – maintains guarantees on outer-loop result



Multifidelity Monte Carlo

leveraging approximate models to reduce the cost of uncertainty quantification



Problem setup:

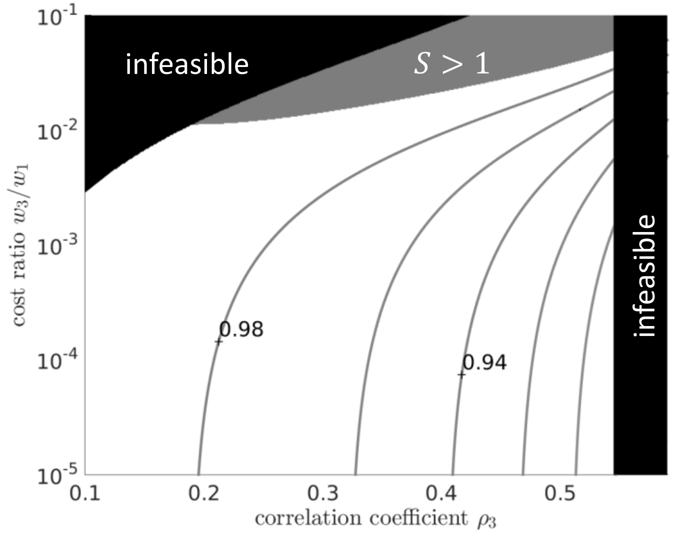
- uncertain input $z \in \mathcal{Z}$, output $y \in \mathcal{Y}$
- high-fidelity model $f^{(1)}: \mathcal{Z} \to \mathcal{Y}$ ("truth")
- $k-1$ surrogate models $f^{(2)}, \dots, f^{(k)}: \mathcal{Z} \to \mathcal{Y}$
- model $f^{(i)}$ has cost w_i
- m_i evaluations for model i, with

$$m_1 \leq m_2 \leq \dots \leq m_k$$

Multifidelity Monte Carlo (MFMC) estimator uses surrogate estimators as control variates:

$$\hat{s} = \bar{y}^{(1)}_{m_1} + \sum_{i=2}^{k} \alpha_i \left(\bar{y}^{(i)}_{m_i} - \bar{y}^{(i)}_{m_{i-1}} \right)$$

- MFMC estimate for the mean ($\hat{s}$)
- mean estimate using m_1 evaluations of truth ($\bar{y}^{(1)}_{m_1}$)
- mean estimate using m_i evaluations of model i ($\bar{y}^{(i)}_{m_i}$)
- mean estimate using m_{i-1} evaluations of model i ($\bar{y}^{(i)}_{m_{i-1}}$)



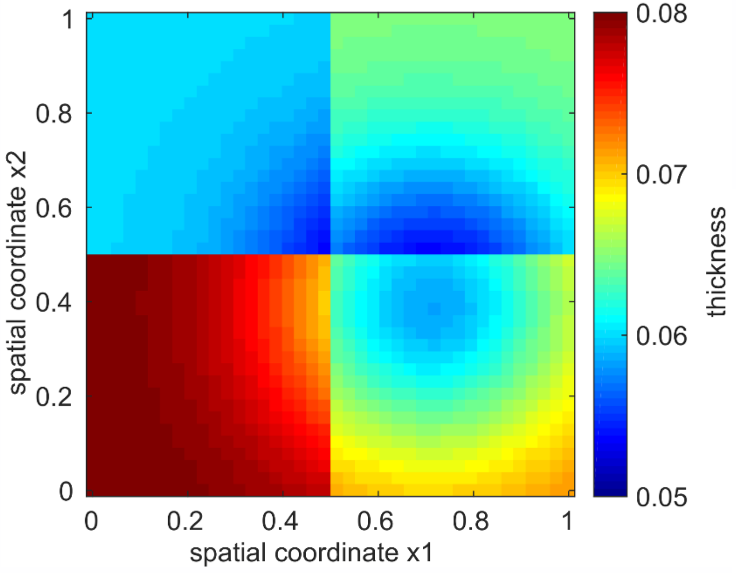
MFMC optimally allocates comp. budget among model evaluations; gives insight to model value relative to model cost.

Multifidelity Monte Carlo

Peherstorfer, Willcox & Gunzburger, "Optimal model management for multifidelity Monte Carlo estimation," *SIAM J. Scientific Computing, 2016*

Example: **structural analysis** of a locally damaged plate

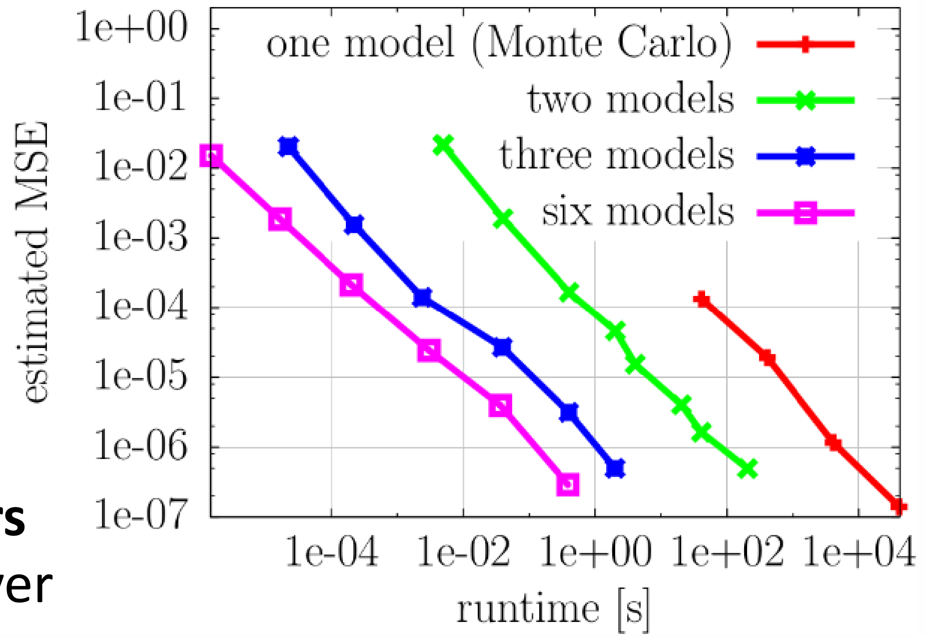
- **Inputs**: nominal thickness, load, two damage parameters
- **Output**: max deflection of plate



- MFMC achieves almost **4 orders of magnitude improvement** over standard Monte Carlo simulation with high-fidelity model only

Six models available

- high-fidelity model FEM, 300 dof
- support vector machine, 256 pts
- POD reduced model, 10 dof
- POD reduced model, 2 dof
- POD reduced model, 5 dof
- data-fit model, linear interpolation, 256 pts



SUMMARY AND CONCLUSIONS

Data to decisions in complex systems: An **offline/online** approach

Offline

- Generate snapshots/libraries, using high-fidelity models
- Generate reduced models

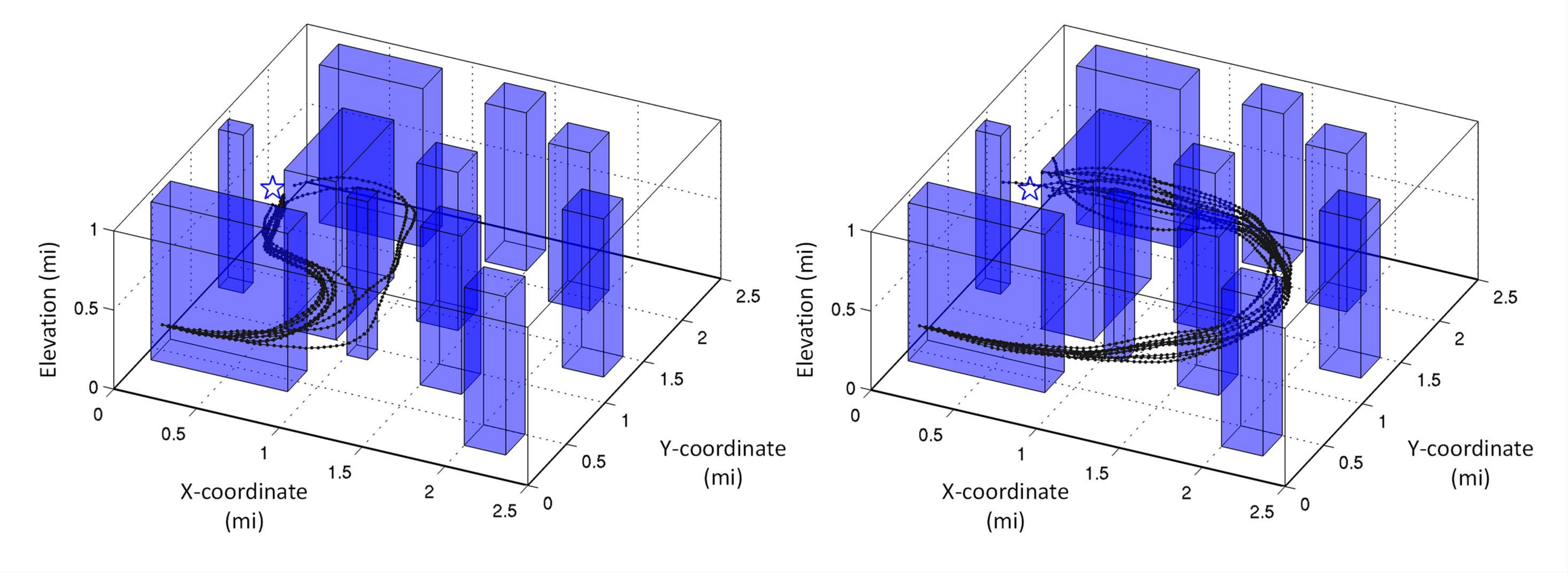
models

Online

- Dynamically collect data from sensors
- Classify system behavior
- Select appropriate library records and/or reduced models
- Rapid {prediction, control, optimization, UQ} using multi-fidelity models
- Adapt reduced models
- Adapt sensing strategies

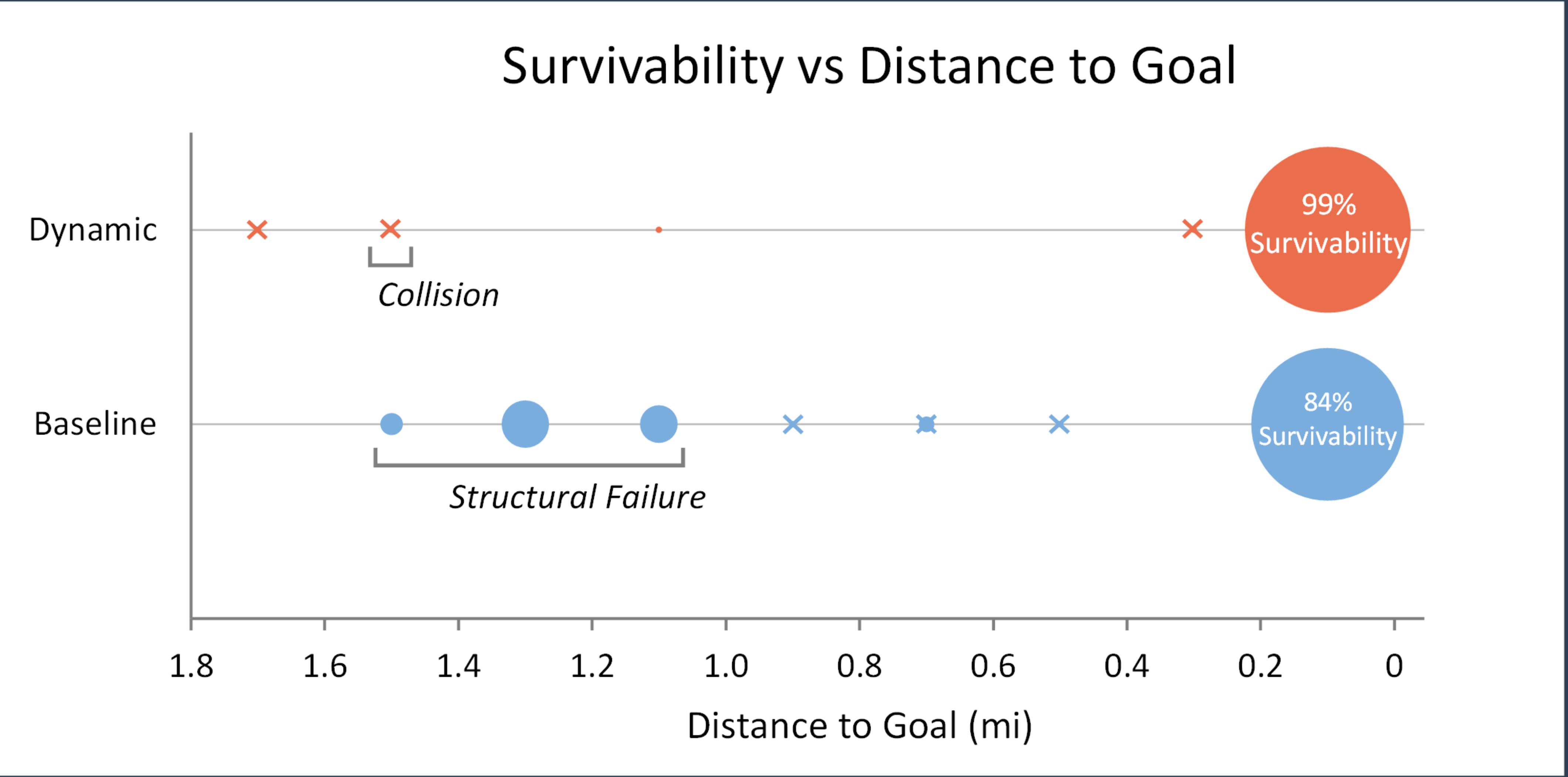
models
+
data

This seems rather more complicated than what I'm doing now onboard my vehicle. *Is it all worthwhile?*



Singh & W., *AIAA J.*, 2017

Data-driven decision-making improves vehicle survivability



Singh & W., *AIAA J.*, 2017

Conclusions

- Reduced models and multifidelity strategies will play an important role in future computational design processes

- Many engineered systems of the future will have abundant sensor data
  → new ways to think about design (digital twin, digital thread)
  → important to leverage the relative strengths of models and data
  → an important role for reduced models, adaptive modeling, multifidelity modeling, uncertainty quantification

Conclusions

But still many important and open challenges:

- scaling from reduced model at panel-level to modeling a complete vehicle (multiscale models, (de)composition)
- managing decision confidence under resource constraints (multifidelity UQ, multifidelity optimization)
- complex nonlinear systems where a local linear subspace approximation is insufficient (nonlinear embeddings)
- sensor placement / sensor acquisition (decision under uncertainty)
- certifying reduced models, dealing with model inadequacy
- and more…

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  QUANTUM Project (F. Fahroo)

- Air Force Dynamic Data-Driven Application Systems (DDDAS) Program (F. Darema)

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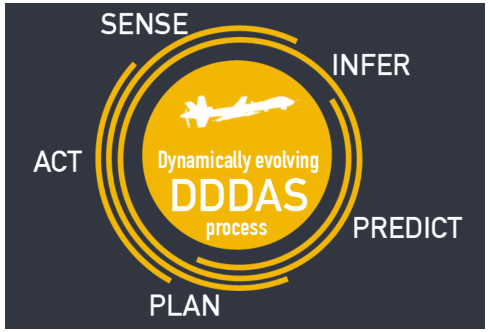

Thank you