UNIVERSITY OF WYOMING

Adjoint based, Error Controlled, Loosely Coupled, Unstructured Design Optimization and Adaptive Mesh Refinement Using FUN3D

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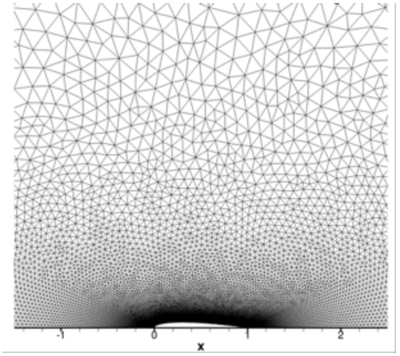
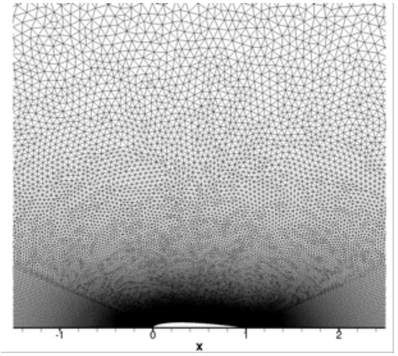
Outline

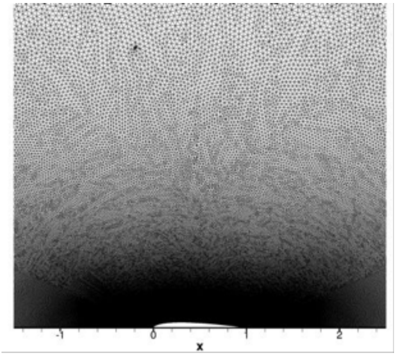
- Introduction
- Motivation & Objective
- Methodology
- Results
- Conclusions
- Recommendations for Future Work

Introduction

- Gradient-Based Design Optimization
  - Current best practices use fixed-complexity meshes
  - Unknown discretization error

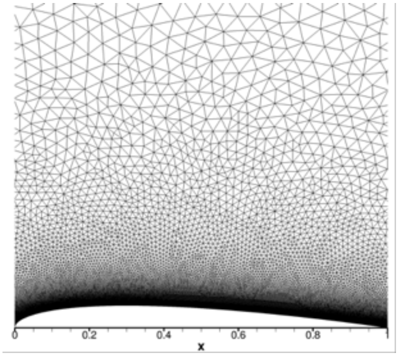


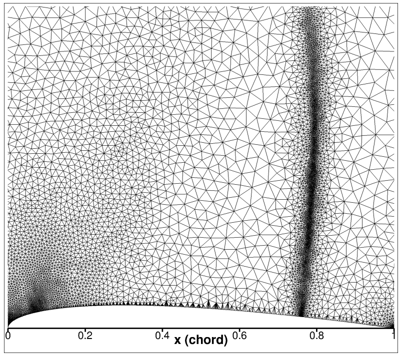




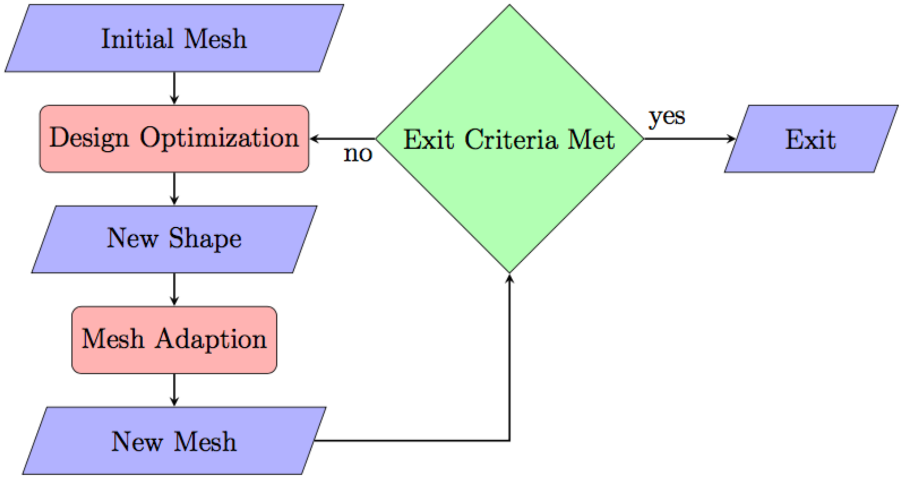
Introduction

- Adjoint-Based Adaptive Mesh Refinement
  - Start with initial mesh with unquantified error
  - Quantify and reduce discretization error related to objective or functional
    - Adjoint provides functional error estimate
      - Used to drive adaption process





Motivation & Objective



- Previous work has combined design optimization and AMR using Finite Element Methods (Li and Hartmann, 2016) and Cartesian Cell Finite Volume Methods (Anderson, 2015)
- This work will combine these two adjoint based features using FUN3D, a Finite Volume, unstructured solver

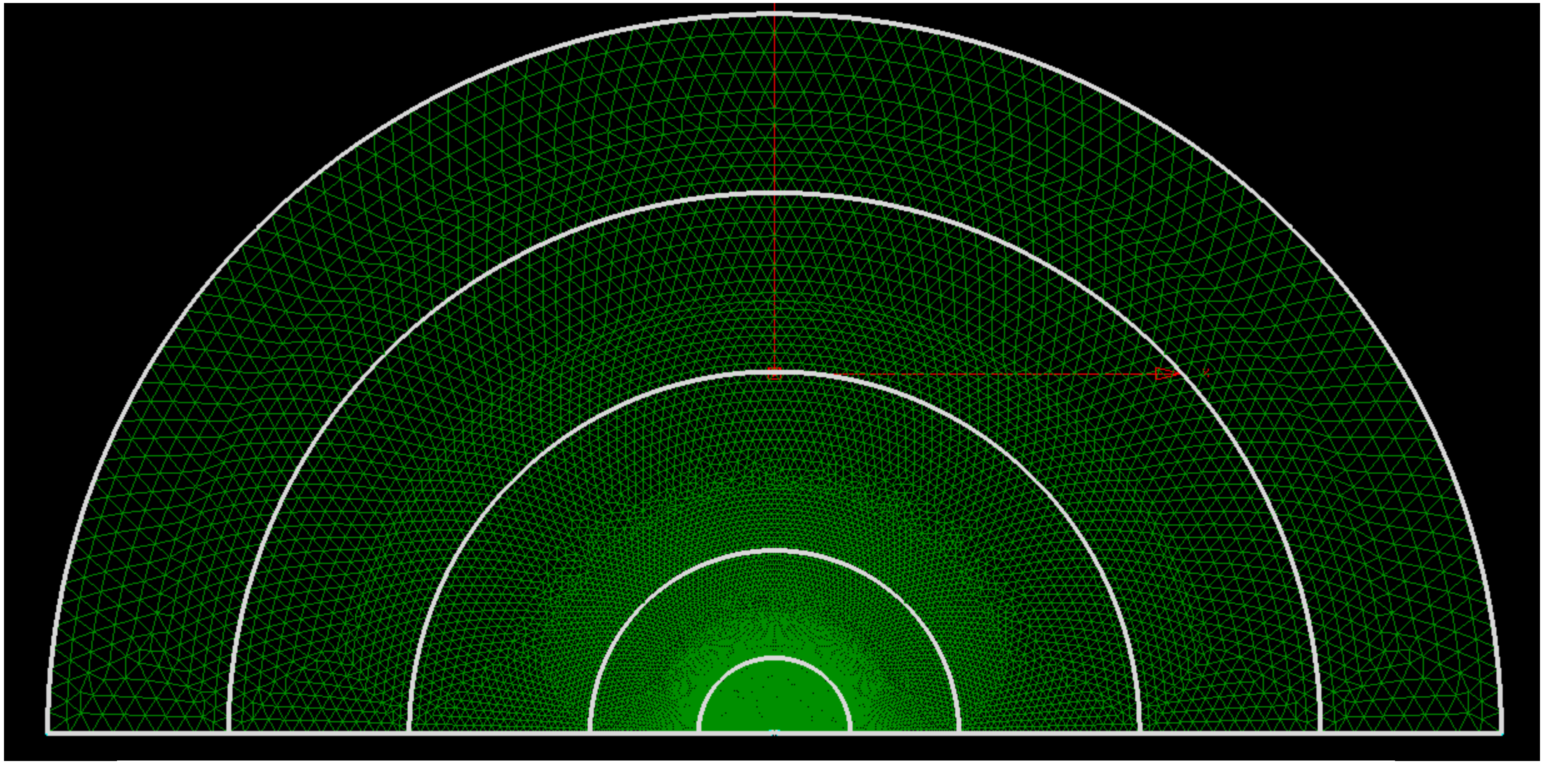
Flow Solver

FUN3D

- 3D unstructured finite-volume RANS solver.
- 2nd –order accurate in space and time.
- Van Leer Flux vector splitting method
- Test Case 1 - Compressible, Euler Equations
  - Adjoint Formulation Available
- Test Case 2 – RANS* using the one equation 1992 Spalart-Allmaras turbulence model
  - Adjoint Formulation Available
- Deforming mesh capability
- Mesh adaption capability

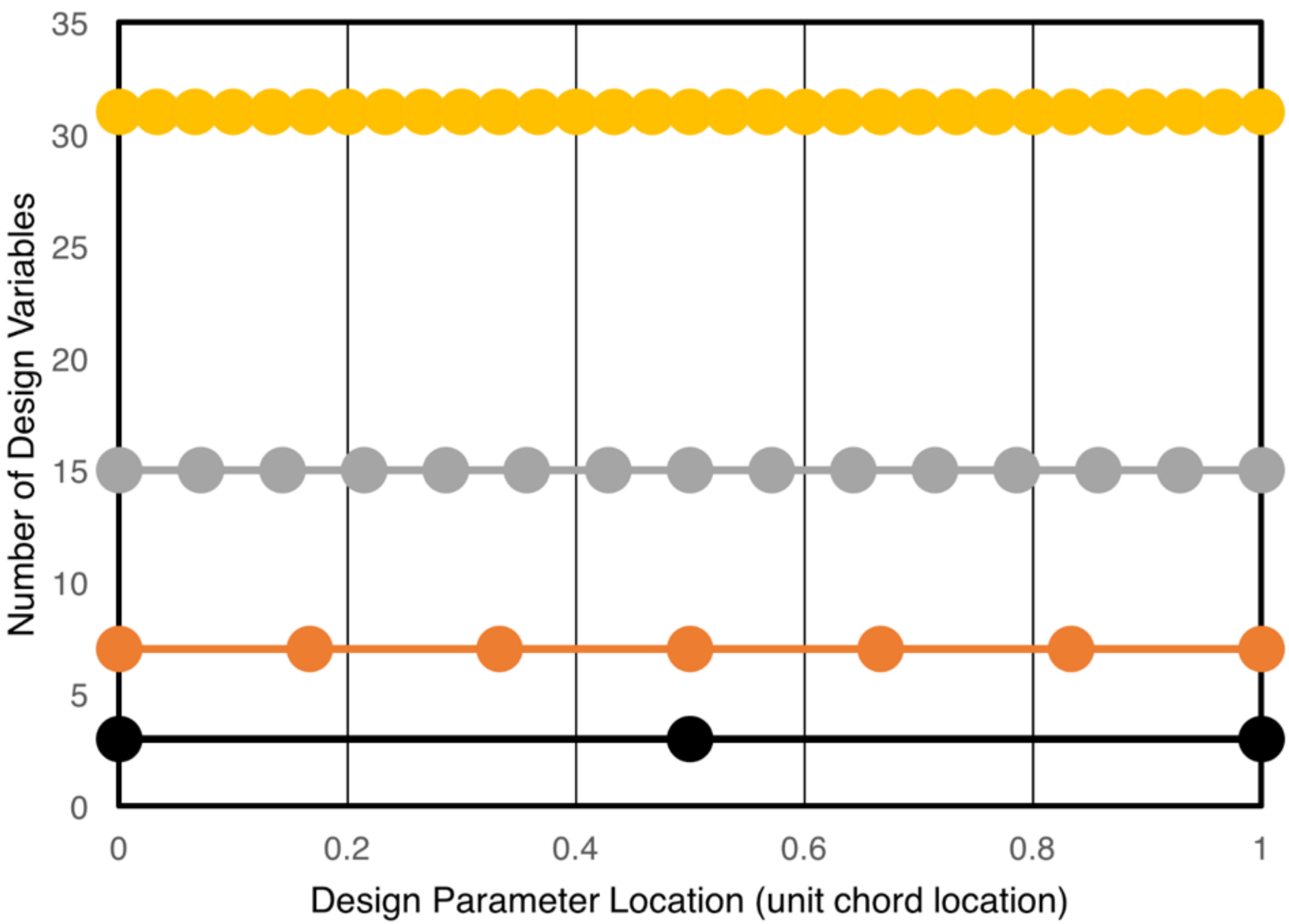
*RANS: Reynolds Averaged Navier-Stokes

Collared Mesh Approach



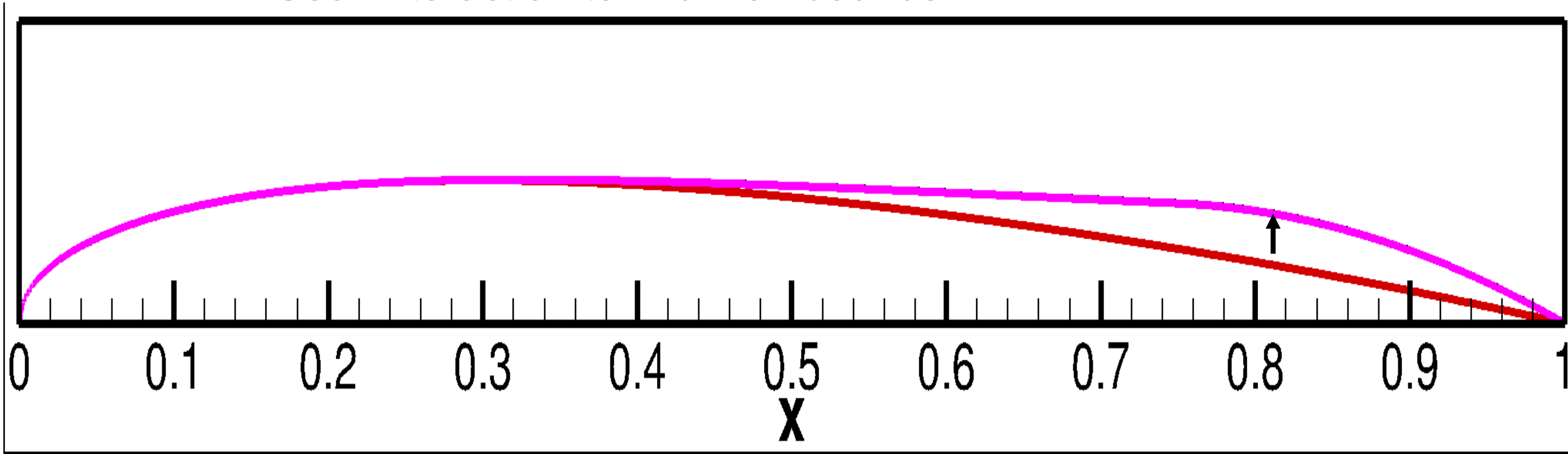
Boundary Distance	Collared Mesh Node Count	Non-Collared Mesh Node Count
5	10752	10752
10	11600	10892
25	12615	11120
50	13514	11229
75	14110	11329
100	14564	11255

Fixed and Progressive Parameterization

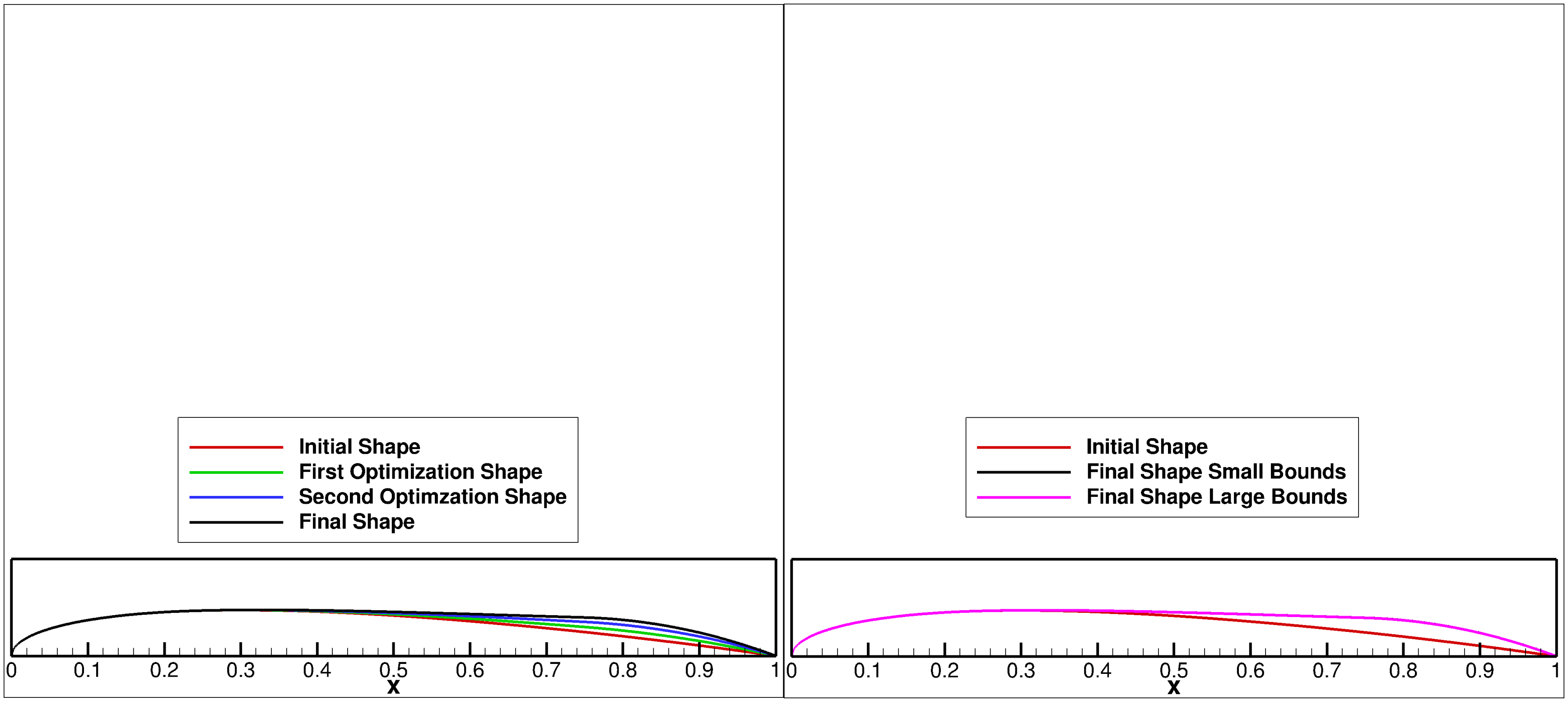


Term Definition

- Bounds
- Reparameterization and effective bounds
- Optimization circuit
- Two approaches for bounds
  - Small bounds with frequent reparameterization
  - Max bounds with fewer reparameterizations
    - User interaction to find max bounds



Design Variable Bounds



Small Bounds

Large Bounds

Objective Function

- Goal of the design
- Penalty Functions
  - Test Case 1: Inviscid, symmetric airfoil
    - $L = Cd$
  - Test Case 2: RANS, lifting airfoil
    - $L = Cd^2 + (0.7 - Cl)2$
- Optimization Complexity Function
  - $O.C.F. = L^2 * \sqrt{0.1 * F.S.C. * 10 * O.C. * 0.01 * M.C.}$
  - F.S.C.: Flow Solver Calls
  - O.C.: Optimization Circuit
  - M.C.: Mesh Complexity
    - Rewards a small objective function but penalizes small objective functions that require increased user interaction and cost of flow solution

Coupling of AMR and Optimization

- Adapt then Design or Design then Adapt
  - Purpose is to determine impacts of the order of workflow on final solution for Finite Volume method
  - Micheletti(2011) performed with Finite Element for an advection-diffusion-reaction equation, reaching same solution for both methods

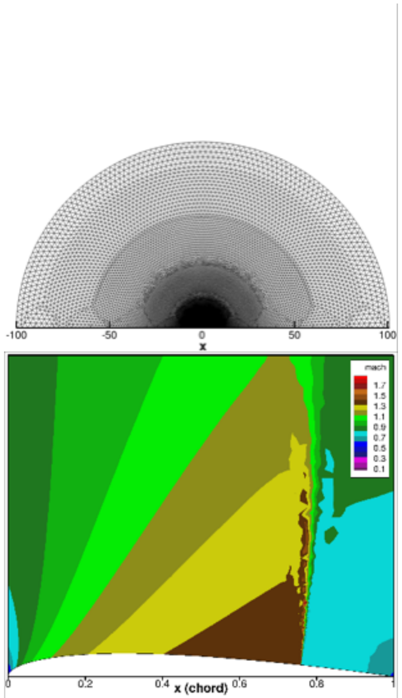
Test Case 1

- NACA-0012
- Inviscid flow
- Mach 0.85, 0^{o} incidence, symmetric airfoil
- Objective: L = Cd
- FUN3D Flow solver using:
  - Van Leer Flux vector splitting
  - Newton Krylov solver with 2500 Krylov Vectors
  - Adjoint computed sensitivities
- SNOPT optimizer (unconstrained)
  - Objective function values
  - Optimality
- Analytic solution is 0 drag (Spalart, 2015)

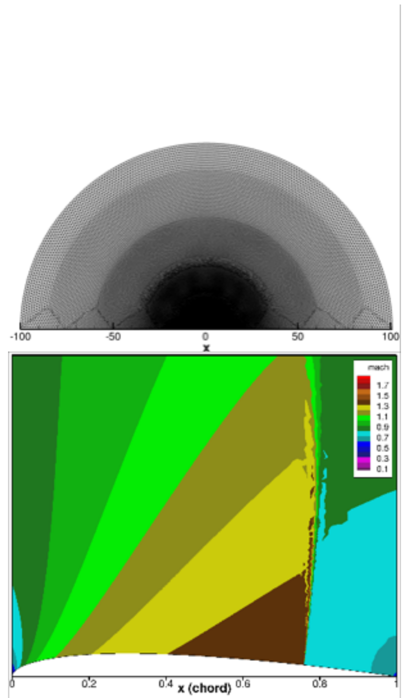
Overview of Test Case 1

- Fixed-complexity meshes
  - Optimization for 3, 7, 15 and 31 design variables
    - Results for first optimization circuit
      - 3 design variables with no reparameterization
    - Fixed bounds and max bounds
      - Multiple optimization circuits
  - Progressive parameterization optimization
- Loosely-coupled AMR and optimization
  - Max bounds for 3, 7, 15 and 31 design variables
  - Progressive parameterization
- Progressive mesh complexity optimization
  - Fixed bounds for 3 and 7 design variables
  - Progressive parameterization for 3, 7 and 15 design variables

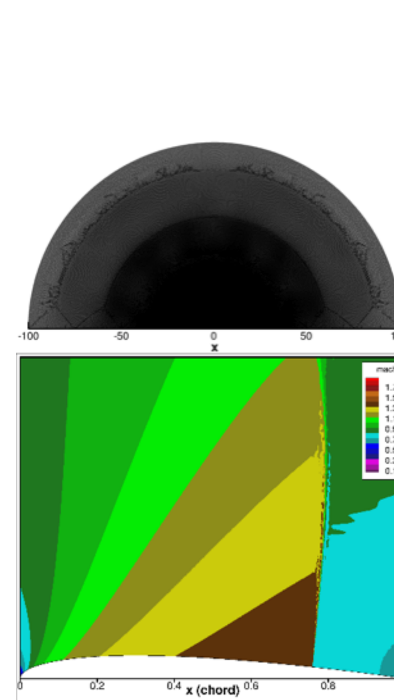
Test Case 1: Initial Conditions



Coarse



Medium

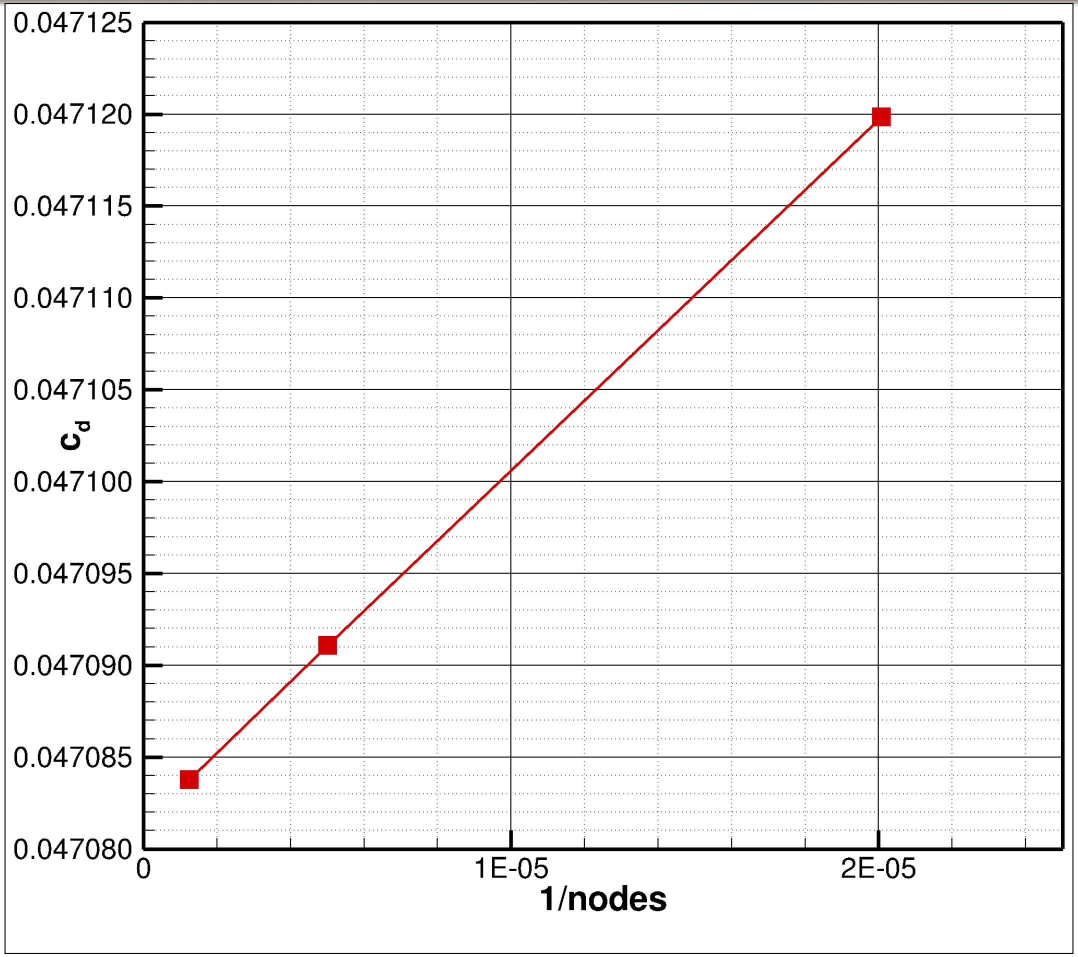


Fine

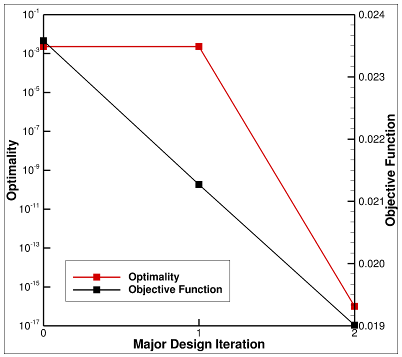
Mesh	Boundary Decay	Airfoil Spacing	Node Count
Coarse	0.9875	0.001	49804
Medium	0.99715	0.0005	199284
Fine	0.99929	0.00025	801073

Test Case 1-Drag Convergence

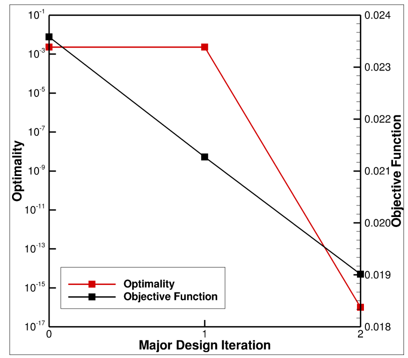
- Slope approximately 2, indicative of
second order accuracy
- Fine mesh value: 470.84
- Infinite value: 470.82



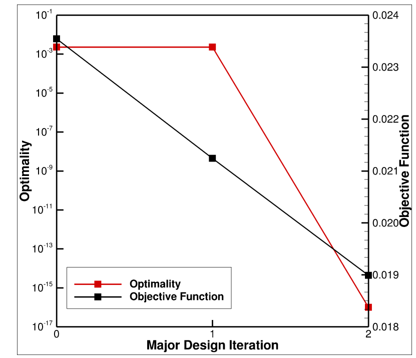
Test Case 1:
3 Design Variables First Optimization Circuit Fixed Parameterization Fixed Bounds



Coarse



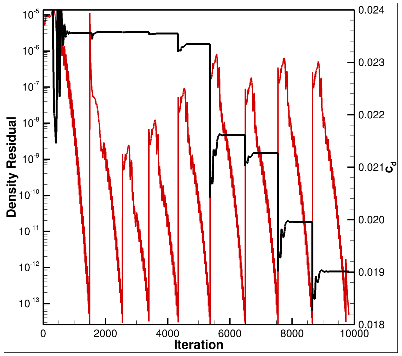
Medium



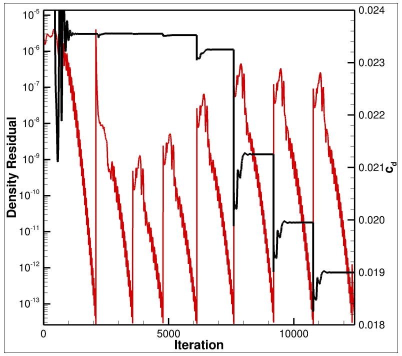
Fine

- Major design iterations performed by SNOPT
  - Include multiple flow solver/adjoint calls
- All meshes reached machine precision for optimality on first optimization circuit
- Multiple circuits performed to converge the objective function, as the design is against the bounds for this optimization circuit

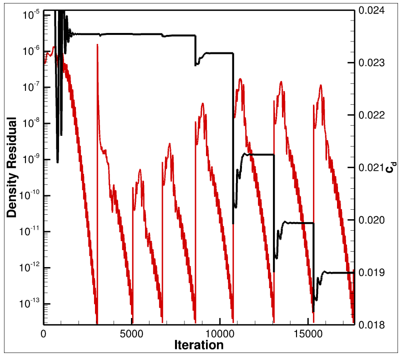
Test Case 1- 3 Design Variables First Optimization Circuit Fixed Parameterization Fixed Bounds



Coarse



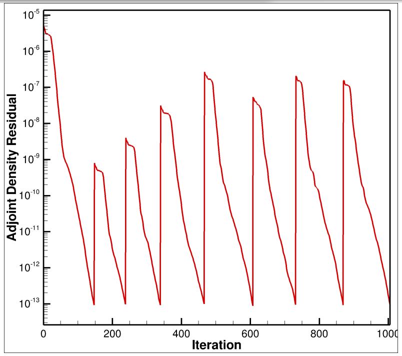
Medium



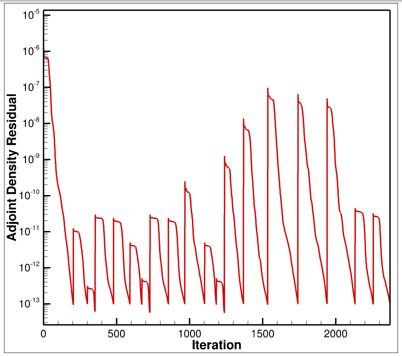
Fine

- Total of 8 flow solver calls by SNOPT for medium and fine mesh
- Total of 9 flow solver calls by SNOPT for coarse mesh
- All flow solver call residuals were converged to 1e-13
- This was a requirement of the design process

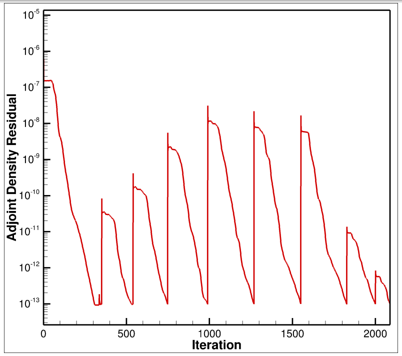
Test Case 1: 3 Design Variables First Optimization Circuit Fixed Parameterization Fixed Bounds



Coarse



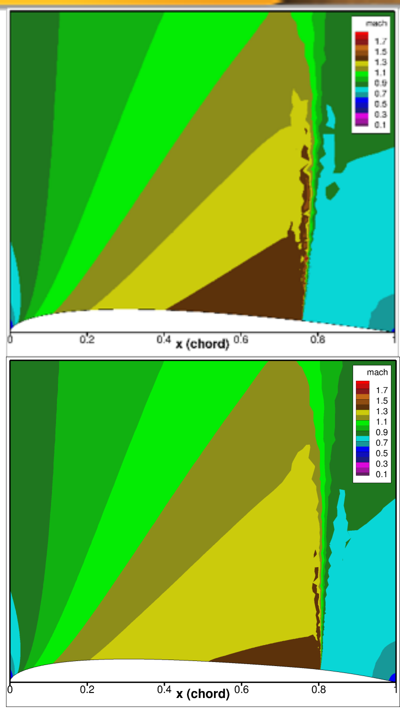
Medium



Fine

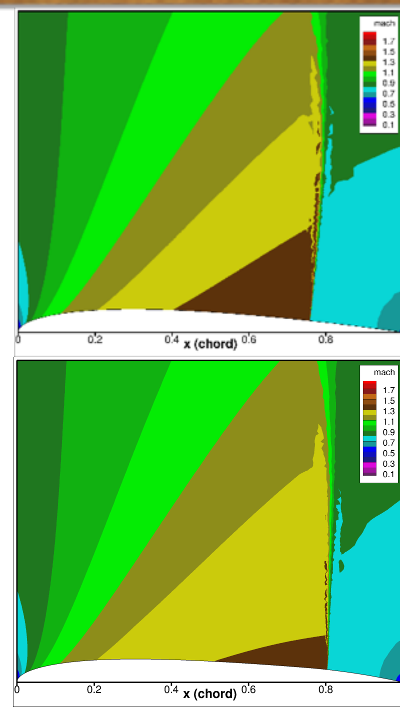
- All adjoint solver call residuals were converged to 1e-13
- This was a requirement of the design process

Test Case 1: 3 Design Variables First Optimization Circuit Fixed-Parameterization Fixed-Bounds



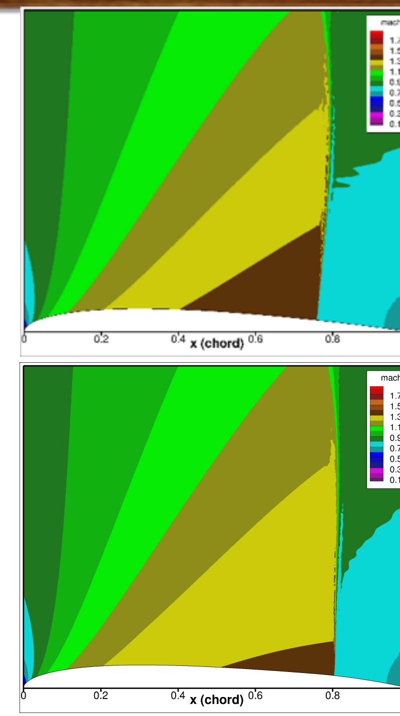
Initial

1st



Initial

1st



Coarse

Medium

Fine

- Shock moves ~ 0.05 chord units
- Drag is reduced by ~ 90 counts

Mesh	Flow Solver Calls	Objective Function: C_d	Optimality	Optimization Complexity Function
Coarse	9	0.038022	1e-16	0.09679
Medium	8	0.037994	1e-16	0.18227
Fine	8	0.037980	1e-16	0.36517

Test Case 1: 3 and 7 Design Variable Final Values Fixed Parameterization Fixed Bounds

- Multiple optimization circuits performed, allowing for larger effective bounds

- 3 DV case able to reduce drag by ~ 180 counts
- Coarse mesh reached machine precision on optimality, used as stopping point for other two meshes
- 7 DV case able to reduce drag by ~ 330 counts
- Coarse mesh reached machine precision on optimality, used as stopping point for other two meshes

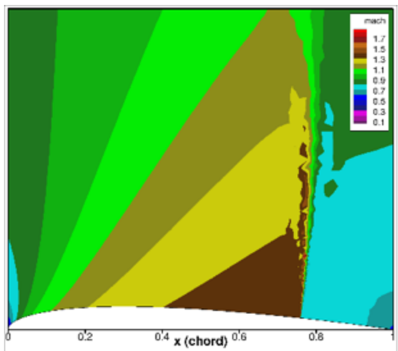
Mesh	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality	Optimization Complexity Function
Coarse	4	48	0.029248	1.0e-16	0.26453
Medium	4	48	0.029194	4.5e-8	0.52720
Fine	4	78	0.029152	7.7e-10	1.34354

3 Design-Variables

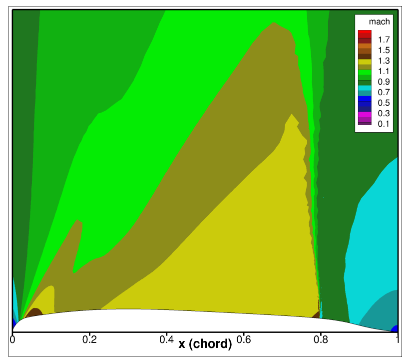
Mesh	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality	Optimization Complexity Function
Coarse	7	221	0.013624	1.0e-16	0.16292
Medium	7	269	0.013606	1.5e-6	0.35861
Fine	7	277	0.013598	1.2e-8	0.72874

7 Design-Variables

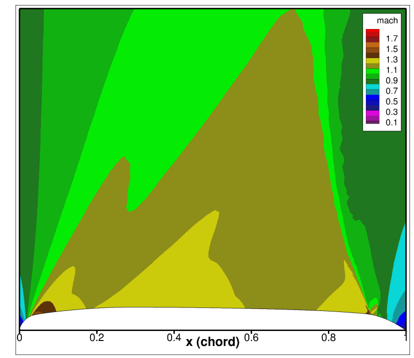
Test Case 1:
15 Design Variables Fixed Parameterization Fixed Bounds



Initial



1st

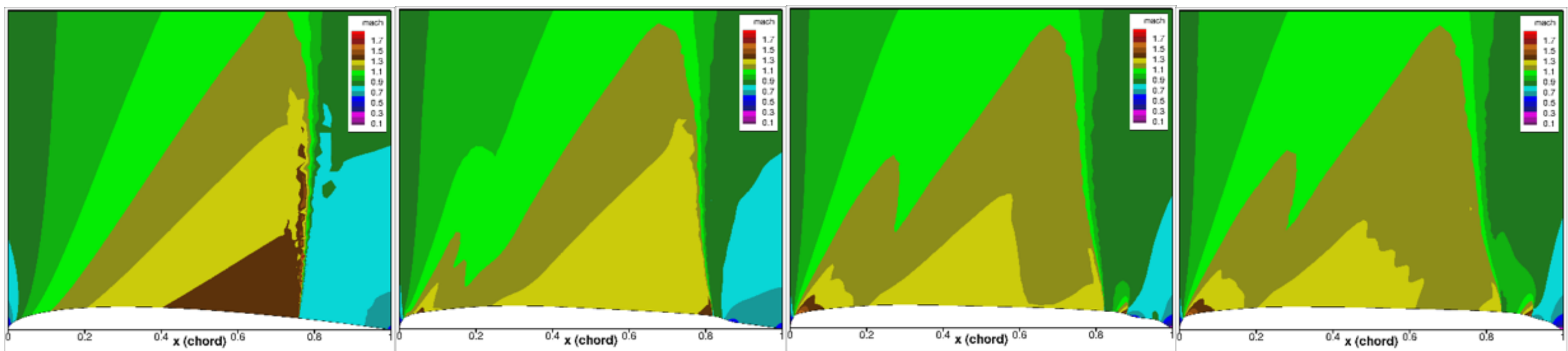


4th

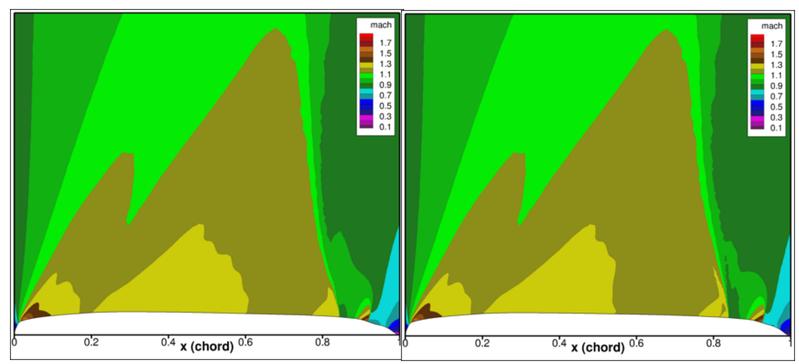
- "Dove-tail" formed, causing increases in drag followed by reductions throughout the optimization circuits
- Only coarse mesh used moving forward unless denoted otherwise
- 15 DV able to reduce drag ~ 410 counts

Optimization Circuits	Flow Solver Calls	Objective Function: C_d	Optimality	Optimization Complexity Function
17	472	0.005898	8.3e-2	0.06954

Test Case 1:
31 Design Variables Fixed Parameterization
Fixed Bounds



Initial | 1^{st} | 9^{th} | 14^{th}



15^{th} | 16^{th}

- "Dove-tail" formed, causing increases in drag followed by reductions throughout the optimization circuits
- 31 DV able to reduce drag ~ 430 counts

Optimization Circuits	Flow Solver Calls	Objective Function: C_d	Optimality	Optimization Complexity Function
28	926	0.004146	7.6e-8	0.06177

Test Case 1 – Fixed Parameterization Max Bounds

- Max bounds able to achieve approximately same values as fixed bounds case at a lower OCF
- Max bounds required much more trial and error to get completed optimization circuit

Design Variables	Optimization Circuits	Flow Solver Calls	Objective Function: C_d	Optimality	Optimization Complexity Function
3	1	21	0.029246	1.7e-5	0.08747
7	1	72	0.013620	4.3e-7	0.03513
15	5	573	0.004044	3.4e-6	0.01954
31	18	677	0.004075	5.1e-8	0.04091

Max Bounds

Design Variables	Optimization Circuits	Flow Solver Calls	Objective Function: C_d	Optimality	Optimization Complexity Function
3	4	48	0.029248	1e-16	0.26453
7	7	221	0.013624	1e-16	0.16292
15	17	472	0.005898	8.3e-2	0.06954
31	28	926	0.004146	7.6e-8	0.06177

Fixed Bounds

Test Case 1: Progressive Parameterization

- Max bounds able to achieve approximately same values as fixed bounds case at a lower OCF for most cases
- Max bounds required much more trial and error to get completed optimization circuit

Design Variables	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality	Optimization Complexity Function
3	1	21	0.029246	1.7e-5	0.08747
7	1	26	0.013646	4.6e-10	0.02119
15	11	364	0.007244	1.7e-5	0.07410
31	7	321	0.003858	5.9e-7	0.01575

Max Bounds

Design Variables	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality	Optimization Complexity Function
3	4	48	0.029248	1.0e-16	0.26453
7	4	216	0.013956	1.9e-5	0.12776
15	6	166	0.006396	1.2e-9	0.002881
31	10	400	0.004442	1.6e-5	0.002785

Fixed Bounds

Test Case 1: Progressive vs Fixed Parameterization

- Max bounds required much more trial and error to get completed optimization circuit

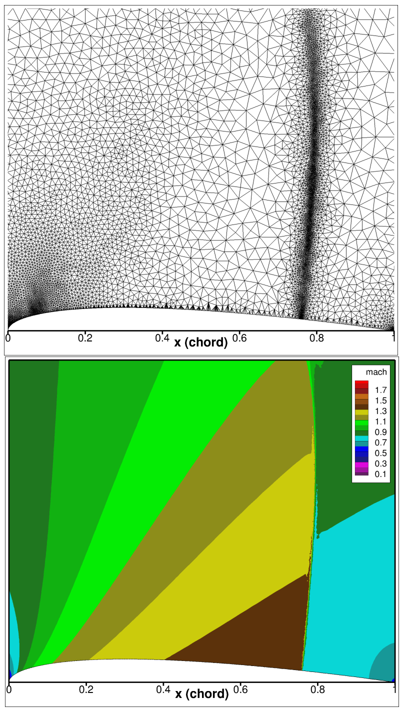
Method	Design Variables	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality
Fixed	31	18	677	0.004075	5.1e-8
Progressive	31	20	732	0.003858	5.9e-7

Max Bounds

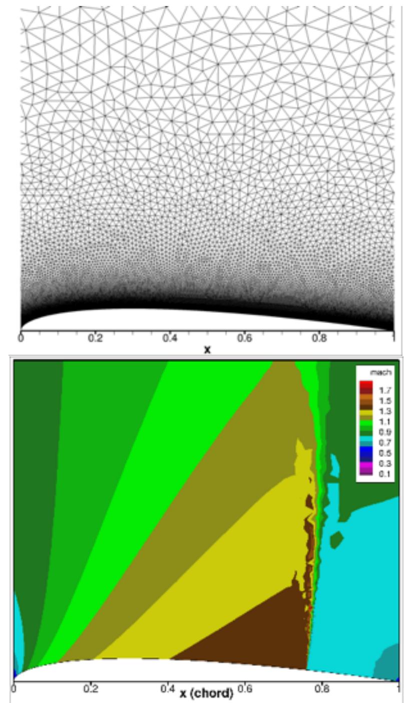
Method	Design Variables	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality
Fixed	31	28	926	0.004146	7.6e-8
Progressive	31	24	830	0.004442	1.6e-5

Fixed Bounds

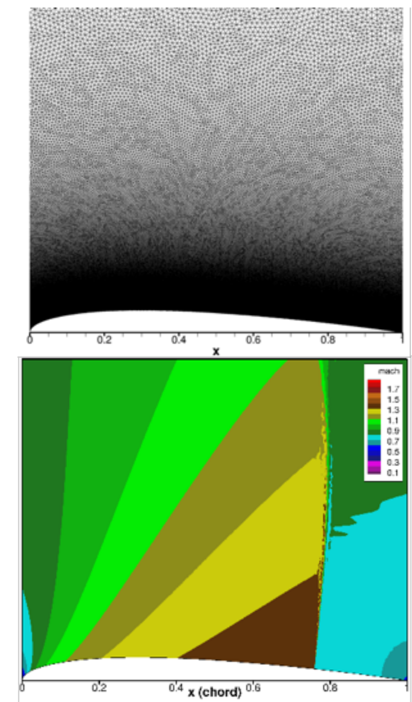
Test Case 1: Adapted Mesh



Adapted
15,000 nodes
Cd = 471.30



Coarse
50,000 nodes
Cd = 471.20

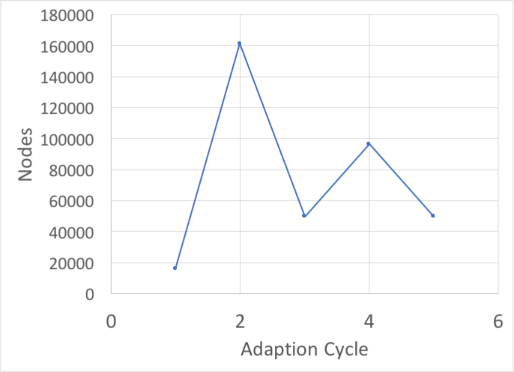


Fine
800,000 nodes
Cd = 470.84

- Adapted mesh has sharper shock line
- Accurate solution with fewer mesh nodes

Test Case 1: Adapted Mesh, Max Bounds

- Able to achieve approximately same values at each design variable step
- Fewer optimization circuits required for more refined parameterizations
- Meshes may increase in size above the final mesh value during adaption process



Design Variables	Optimization Circuits	Flow Solver Calls w/ AMR	Flow Solver Calls w/o AMR	Mesh Size (nodes)	Objective Function: Cd	Optimality
3	3	75	45	30515	0.029178	1.9e-11
7	6	301	241	32072	0.013600	7.6e-9
15	4	278	238	36964	0.007478	1.6e-5
31	4	542	502	49591	0.004264	1.3e-7

Adapted

Design Variables	Optimization Circuits	Flow Solver Calls	Mesh Size (nodes)	Objective Function: Cd	Optimality
3	1	21	50000	0.029246	1.7e-5
7	1	72	50000	0.013620	4.3e-7
15	5	573	50000	0.004044	3.4e-6
31	18	677	50000	0.004075	5.1e-8

Max Bounds, Coarse Mesh

Test Case 1: Adapted Mesh, Progressive Parameterization

- Able to achieve approximately same values as fixed complexity mesh
- 31 DV case not completed for progressive parameterization as the mesh grew to approximately 1,000,000 nodes.

Design Variables	Optimization Circuits	Flow Solver Calls w/ AMR	Flow Solver Calls w/o AMR	Mesh Size (nodes)	Objective Function: Cd	Optimality
3	3	75	45	30515	0.029178	1.9e-11
7	2	155	135	30502	0.013944	5.2e-6
15	8	373	293	56998	0.007738	4.1e-8

Adapted

Design Variables	Optimization Circuits	Flow Solver Calls	Mesh Size (nodes)	Objective Function: Cd	Optimality
3	1	21	50000	0.029246	1.7e-5
7	1	26	50000	0.013646	4.6e-10
15	11	364	50000	0.007244	1.7e-5
31	7	321	50000	0.003858	5.9e-7

Fixed-Complexity Coarse mesh

Test Case 1 - Adapt then Design vs Design then Adapt

- Achieved approximately same solution
- Design then AMR achieved this sooner
- Final shapes are the same

Method	Optimization Circuits	Flow Solver Calls	Mesh Size (nodes)	Objective Function: Cd	Optimality
AMR then Design	6	301	32072	0.013600	7.6e-9
Design then AMR	4	172	35198	0.013680	1.7e-5

Test Case 1:
Progressive Mesh, Fixed Parameterization

- Slight improvements seen during the progressive mesh design for the fixed parameterization
- Could be useful for complex 3D geometries and for fine tuning
- Reduced total flow solver calls required for Fine mesh computations

Mesh	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality	Optimization Complexity Function
Coarse	1	21	0.029247	1.7e-5	0.08748
Medium	1	22	0.029193	2.6e-8	0.17845
Fine	1(3)	11 (54)	0.029151	2.0e-6	0.25225
Fine-Fixed	4	78	0.029152	7.7e-10	1.34354

3 Design-Variables

Mesh	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality	Optimization Complexity Function
Coarse	1	72	0.013621	4.3e-7	0.03513
Medium	1	52	0.013604	2.6e-6	0.05958
Fine	1(3)	41 (165)	0.013598	3.7e-6	0.10597
Fine-Fixed	7	277	0.013598	1.2e-8	0.72874

7 Design-Variables

Test Case 1: Progressive Mesh, Progressive Parameterization

- Larger improvement seen over fixed parameterization case
- Can be used on adapted meshes with ever decreasing discretization tolerance
- Able to perform 15 design variable on Fine mesh, as it was too expensive before
- Combining the two makes it even better

Mesh	Optimization Circuits	Flow Solver Calls	Objective Function: Cd	Optimality	Optimization Complexity Function
Coarse	1	21	0.029247	1.7e-5	0.08748
Medium	1(2)	41 (62)	0.013635	1.1e-6	0.05314
Medium-Fixed	7	269	0.013606	1.5e-6	0.35861
Fine	1(3)	33 (95)	0.012148	2.3e-7	0.07588

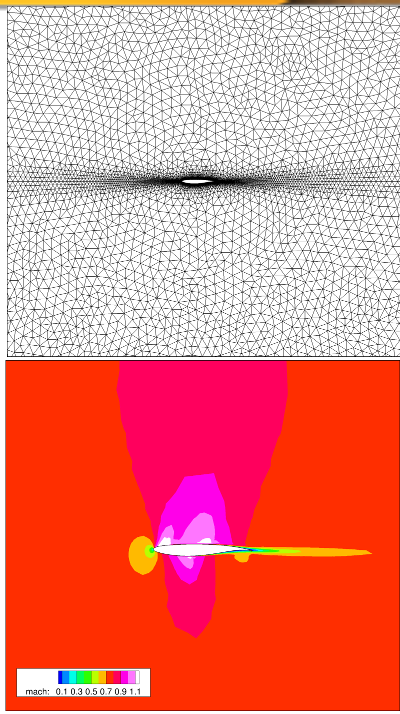
Test Case 2

- TMA-0712, 12% thick, design C_l 0.7, design Mach 0.78
- Viscous flow
- Mach 0.78, 0^o incidence, lifting airfoil
- Objective: $L = C_d^2 = (0.7-C_l)^2$
- FUN3D Flow solver using:
  - Van Leer Flux vector splitting
  - Newton Krylov solver with 1000 Krylov Vectors
  - Adjoint computed sensitivities
- SNOPT optimizer (unconstrained penalty function)
  - Objective function values
  - Optimality

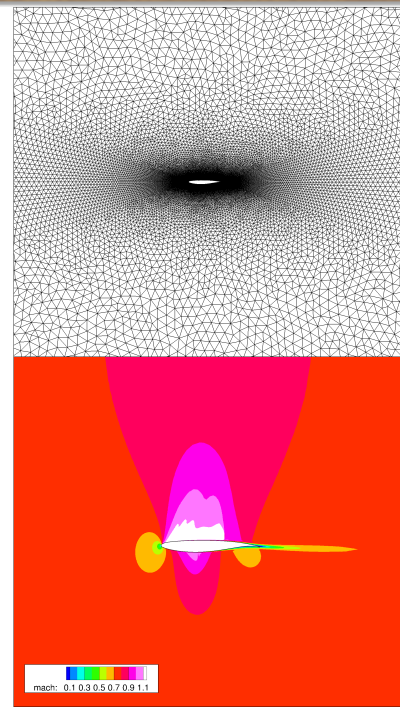
Overview of Test Case 2

- Fine mesh
  - Optimization for 7 design variables
    - Fixed bounds
      - Multiple optimization circuits
  - Progressive parameterization optimization
- Loosely-coupled AMR and optimization
  - Unable to be performed due to isotropic mesh requirement
- Progressive mesh complexity optimization
  - Unable to be performed as the coarse and medium mesh were under resolved and could not provide feasible design

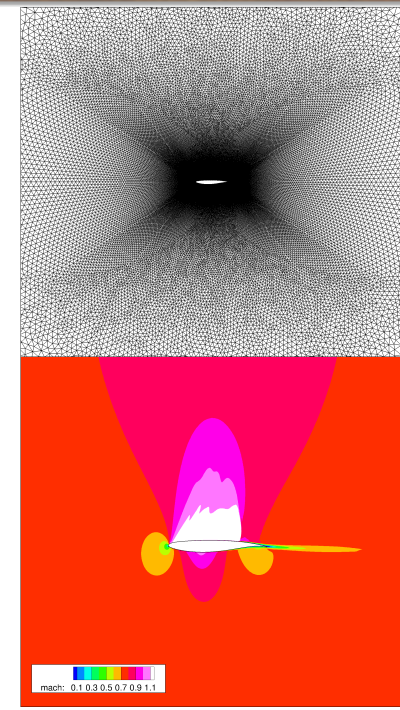
Test Case 2: Initial Conditions



Coarse



Medium

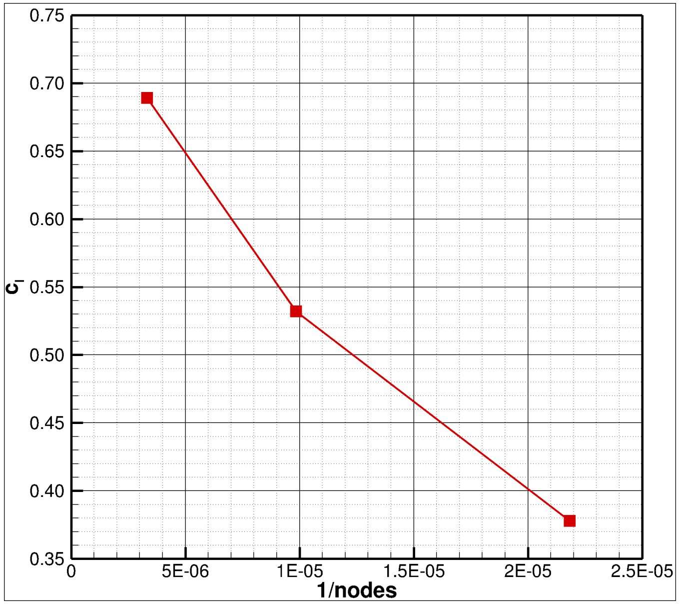


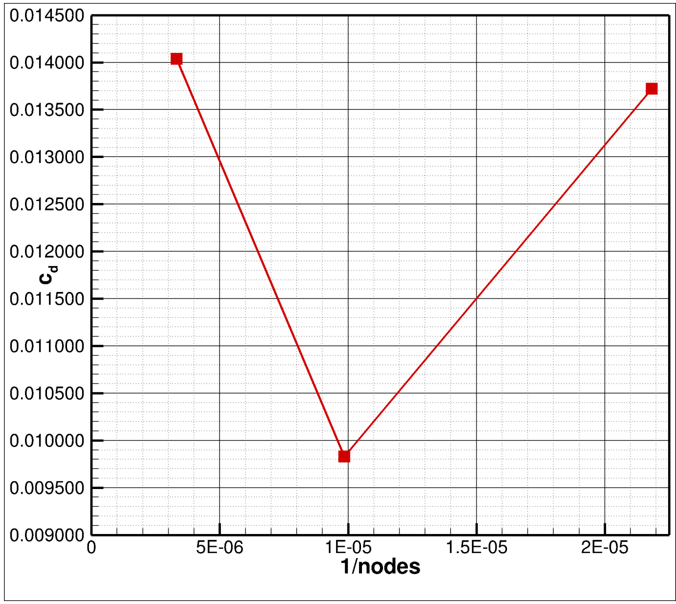
Fine

Mesh	Boundary Decay	Airfoil Spacing	Node Count	Viscous Layers	y+	Growth Rate
Coarse	0.5	0.004	45819	40	3	1.2
Medium	0.982	0.002	101520	40	1.5	1.2
Fine	0.99815	0.001	300782	39	1	1.2

Test Case 2: Drag and Lift Convergence

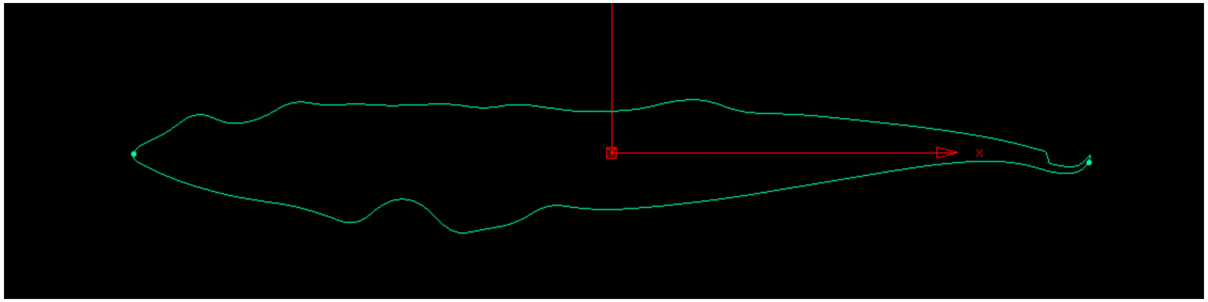
- Fine lift value: 0.6816
- Fine drag value: 0.0134
- Infinite values not realistic as the mesh family does not provide proper convergence





Test Case 2

- The coarse and medium meshes were under refined and lead to infeasible design shapes
- The fine mesh performed well, and was able to provide reasonable initial designs



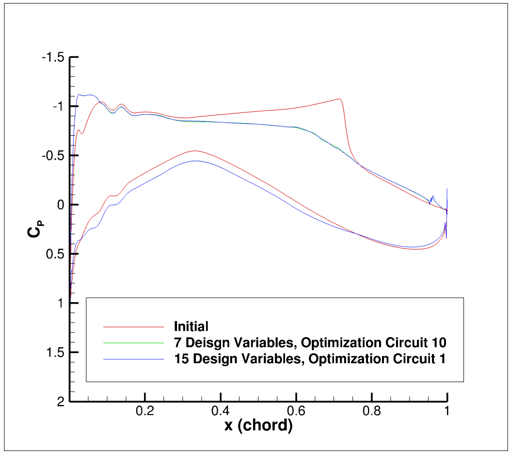
Test Case 2:
7 Design Variables

- 7 thickness and 7 camber design variables were used for this design
- Multiple optimization circuits were used
- Drag reduced by 55 counts
- Lift increased by 0.0178

Mesh	Optimization Circuits	Flow Solver Calls	Lift	Drag	Objective Function	Optimality
Fine	10	1313	0.6994	0.007908	0.0006257	2.2e-6

Test Case 2: Progressive Parameterization

- 7 design variables, followed with 15 design variables
- Minor improvements made to both lift and drag, with lift reaching the target goal



Design Variables	Optimization Circuits	Flow Solver Calls	Lift	Drag	Objective Function	Optimality
7	10	1313	0.6994	0.007908	0.0006257	2.2e-6
15	1	19	0.7000	0.007904	0.0006247	9.1e-5

Test Case 2: Adaptive Mesh Refinement

- Unable to resolve an adapted mesh using isotropic mesh adaption
  - Required surface resolution for unit length airfoil with a Reynolds Number of 30 million is ~2,000,000 nodes

Conclusions

- More design variables lead to more optimal shape
- Number of flow solver calls increases with number of design variables
- Number of flow solver calls increases with mesh complexity
- Bound settings can dramatically impact total cost
- Progressive parameterization reduces number of flow solver calls and cost of optimization
- Progressive mesh complexity reduces number of flow solver calls
- Combination of the two further reduces cost
- AMR with opt reduces cost through mesh complexity reduction
  - Cost increases due to additional flow solver calls for AMR, and user interaction doubled for each optimization circuit

Operational Lessons Learned

- Fully converge flow and adjoint solutions
  - Prevents stalling of optimization problem
- Fully converge mesh movement problem
  - Necessary to avoid negative cell volumes
- Tightening bounds
  - Bounds can have significant effect on optimization progress and results
- Isotropic mesh adaption
- Restart vs freestream start
  - Convergence issues observed with freestream initializations
- Bounds on adapted meshes
  - Intermediate irregular shapes lead to large mesh sizes

Future Work

- Progressive AMR error tolerances
- Anisotropic mesh adaption
- Global design searches
- Uncertainty quantification
- Coupling FUN3D design and adaption framework
- Three-dimensional

Questions?