Development of a High-Fidelity Aero-Thermo-Elastic Analysis and Design Capability

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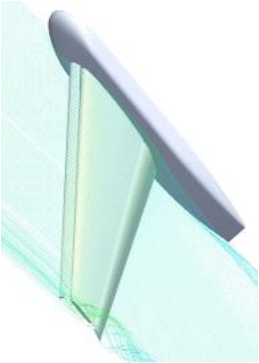
OUTLINE

- Introduction
  - Background
  - Challenges
  - Objectives
  - Outline of Project
- Aero-Thermo-Elastic Coupling Description
- Analysis Results
- Sensitivity Analysis and Optimization
- Sensitivity Analysis and Optimization Results
- Conclusions and Future Works

INTRODUCTION
Background

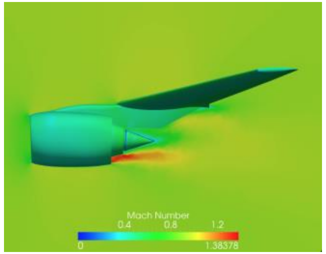
Why use numerical analysis?

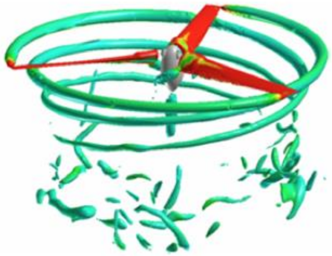
- ✓ Relatively low cost
- ✓ Speed



Why use multi-disciplinary computation?

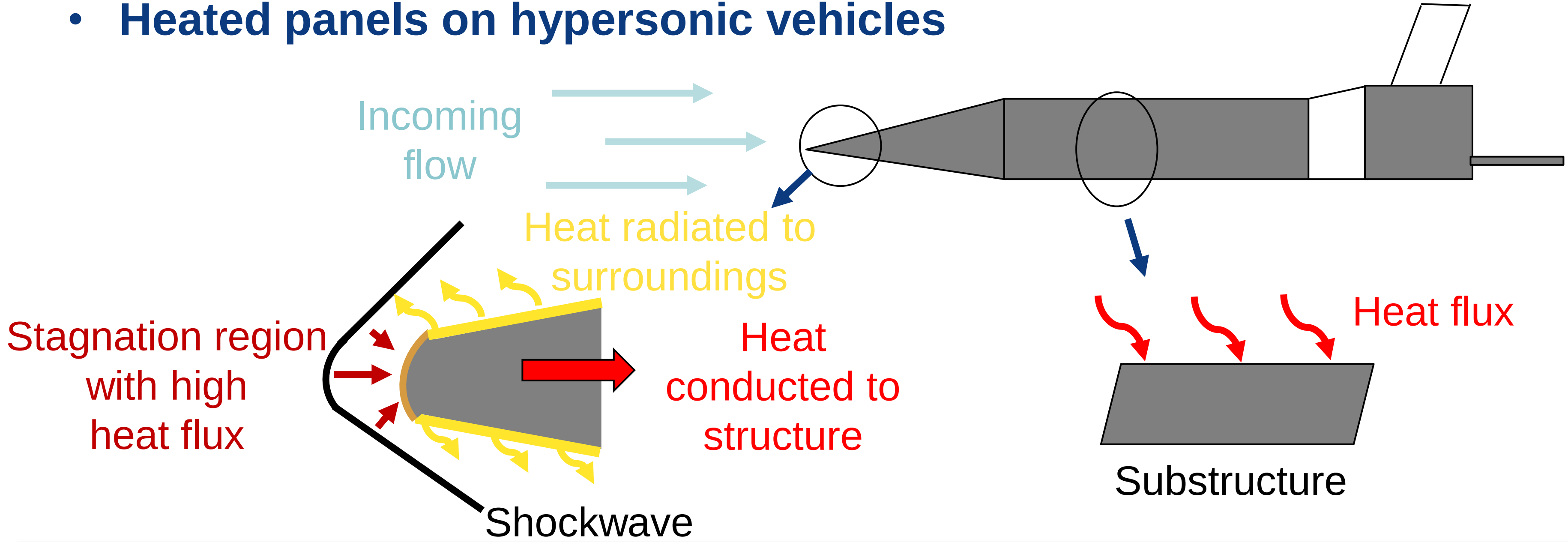
- ✓ Allows complimentary information
- ✓ Adds to accuracy of results
- ✗ Adds to the complexity





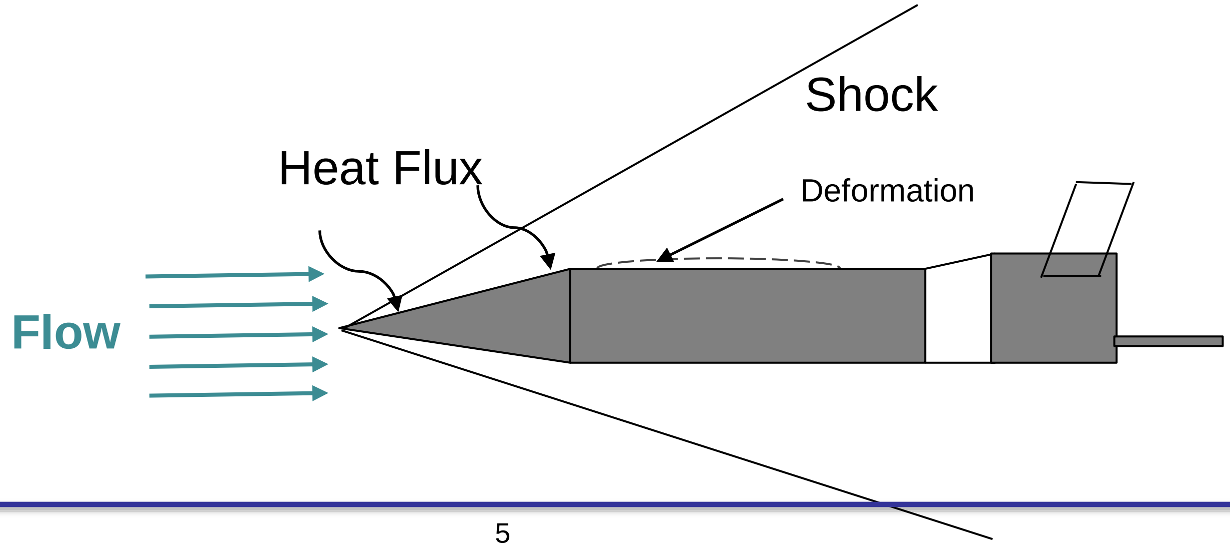
➢ One of the major applications of **aero-thermo-elastic simulations** is in **Hypersonic vehicles:**

- **Leading edges of hypersonic vehicles**
- **Heated panels on hypersonic vehicles**



INTRODUCTION
Background

- Accurate aero-thermo-elastic analysis and design requires:
  - ✓ Aerodynamic loads (aerodynamic pressure and viscous forces)
  - ✓ Aero-thermal effects (surface heating rate and inner temperature distributions)
  - ✓ Structural loads (structural deformation and stresses)



Background

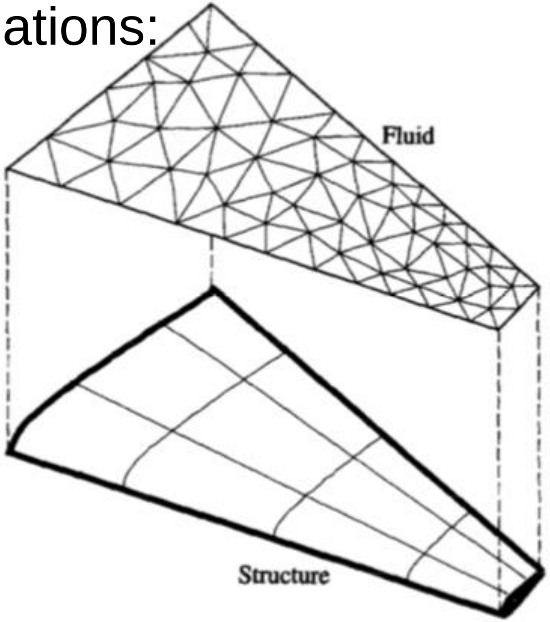
- Two main ways to approach multi-disciplinary simulations:

- Strong coupling
  - ✓ More stable approach
  - ✗ Cannot use already available and well-tested solvers

- Weak coupling
  - ✓ Able to use existing well-developed and tested codes
  - ✗ Less stable

INTRODUCTION
Challenges

- Main challenges in coupled aero-thermo-elastic simulations:
  - ✗ Difference in space scale
  - ✗ Difference in time scale
  - ✗ Dealing with the boundary conditions
  - ✗ Coupling the sensitivities



Non-matching Fluid/Structure interface*

*Farhat et al, Load and Motion Transfer Algorithm for Fluid/Structure Interaction Problems with Non-Matching Discrete Interfaces (1998).

INTRODUCTION
Objectives

Develop/Validate a **coupled aero-thermo-elastic** analysis and design capability which:

- ✓ Uses **weak coupling** in order to take advantage of the already available and well tested in-house codes.
- ✓ Uses **high-fidelity** models for each discipline.
- ✓ Performs **transient** analysis in **3D.**
- ✓ Performs **Tangent and Adjoint sensitivity analysis**.

INTRODUCTION
Outline of Project

- Analysis
  - ✓ Validate the **thermal analysis** capability **.**
  - ✓ Validate the **thermo-elastic analysis** capability.
  - ✓ Develop/Validate **aero-thermo-elastic analysis** capability.

- Design Optimization
  - ✓ Verify the **thermo-elastic adjoint sensitivities**.
  - ✓ Demonstrate standalone **thermo-elastic optimization**.
  - ✓ Develop/Verify **aero-thermo-elastic adjoint sensitivities.**
  - ✓ Demonstrate **aero-thermo-elastic Optimization.**

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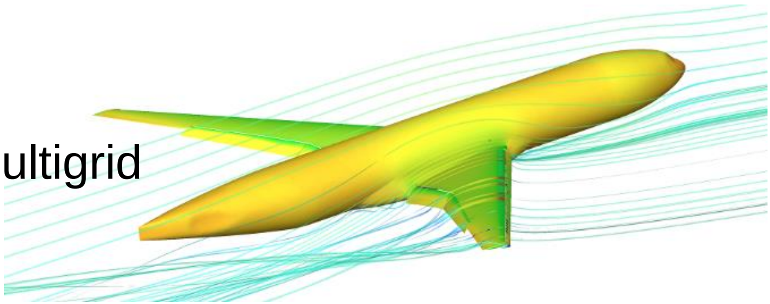
- Aero-Thermo-Elastic Coupling Description
  - Flow Solver with Mesh Deformation Capability
  - Structural Solver
  - Fluid-Structure Interaction (FSI) Module
- Analysis Results
- Sensitivity Analysis and Optimization
- Sensitivity Analysis and Optimization Results
- Conclusions and Future Works

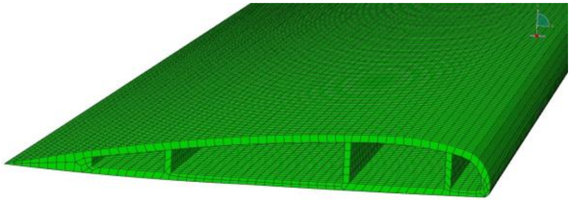
AERO-THERMO-ELASTIC COUPLING
Flow Solver with Mesh Deformation Capability

- Flow Solver: Navier-Stokes Unstructured 3D **(NSU3D)**
  - ✓ Based on the conservative form of the Navier-Stokes: $\frac{\partial u(x,t)}{\partial t} + \nabla . F(u) = 0$
  - ✓ 3D unstructured finite-volume RANS solver
  - ✓ Vertex-centered
  - ✓ 2^{nd} order accurate in space and time
  - ✓ Uses a line-implicit solver with agglomeration multigrid
  - ✓ Fluxes are calculated using the Roe Scheme
  - ✓ **Mesh deformation capability based on the linear elasticity model**
  - ✓ Numerous simulations and participations: **DPW, HiLiftPW, AePW**



AERO-THERMO-ELASTIC COUPLING
Structural Solver

- Structural Solver: Adjoint-based Structural Optimizer **(AStrO)**
  - ✓ High-fidelity, open-source, developed in-house
  - ✓ Finite-element modeling of 3D structures
  - ✓ Compatible with Abaqus input and output files
  - ✓ Static and dynamic analysis:
    - ✓ Elasticity problem: $\nabla \cdot \sigma - \xi \frac{du}{dt} - \rho \frac{d^2u}{dt^2} + f = 0 \Rightarrow [K]U + [C]\dot{U} + [M]\ddot{U} = F$
    - ✓ Heat transfer problem: $\rho c \frac{\partial T}{\partial t} + \nabla \cdot (k \nabla T) - Q = 0 \Rightarrow [K_{therm}]T + [M_{therm}]\dot{T} = F_{therm}$
    - ✓ Thermo-elastic problem
  - ✓ Time stepping with Newmark-β expansion



AERO-THERMO-ELASTIC COUPLING
Structural Solver

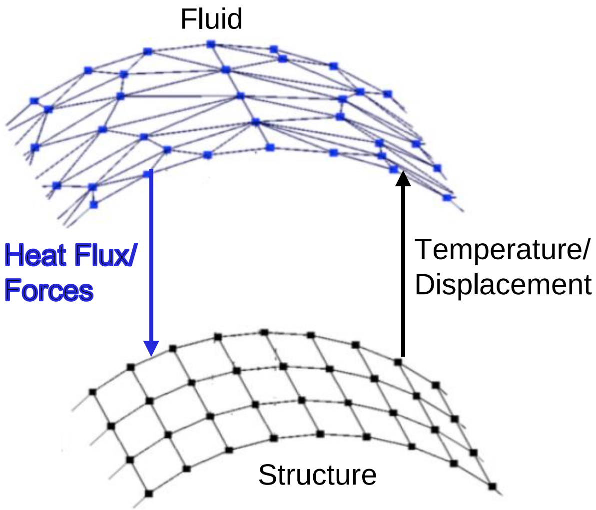
- Assumptions made for **thermo-elastic** coupling in **AStrO**:
  - ✓ Thermal material properties have no significant dependence on strain.
  - ✓ The heat generated by deformation is assumed to be negligible.

➡ Deformation has a one-way dependence on the temperature distribution.

➡ The effect of thermal expansion shows up as part of the load in the elasticity equation.

AERO-THERMO-ELASTIC COUPLING
Fluid-Structure Interaction (FSI) Module

- **Weak coupling** requires :
  - ✓ convergence of the following at the boundary:
    - ✓ Temperature
    - ✓ Heat flux
    - ✓ Aerodynamic loads
    - ✓ Displacements
  - ✓ **FFTB** method or **Dirichlet-Neumann** boundary conditions for stability and convergence.



Fluid/structure interface boundary conditions*

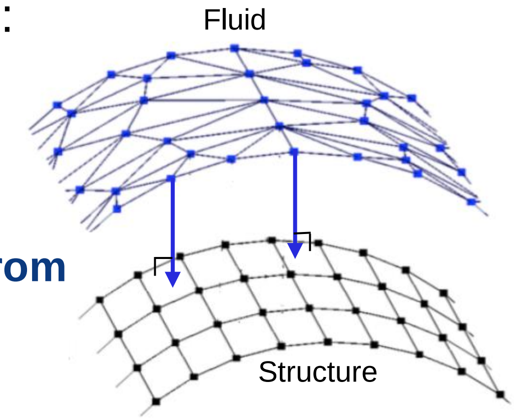
*Li et al, 3D common-refinement method for non-matching meshes in partitioned variational fluid-structure analysis (2017)

AERO-THERMO-ELASTIC COUPLING
Fluid-Structure Interaction (FSI) Module

- The transfer of data between meshes can be summarized as:

$$\begin{cases} Q_{CTSD} = [P]Q_{CFD} \\ T_{CFD} = [P]^{T}T_{CTSD} \end{cases}, \quad \begin{cases} F_{CTSD} = [P]F_{CFD} \\ U_{CFD} = [P]^{T}U_{CTSD} \end{cases}$$

Search algorithm locates nearest/perpendicular projected point from CFD grid point to structure mesh surface.



Fluid/structure data transfer*

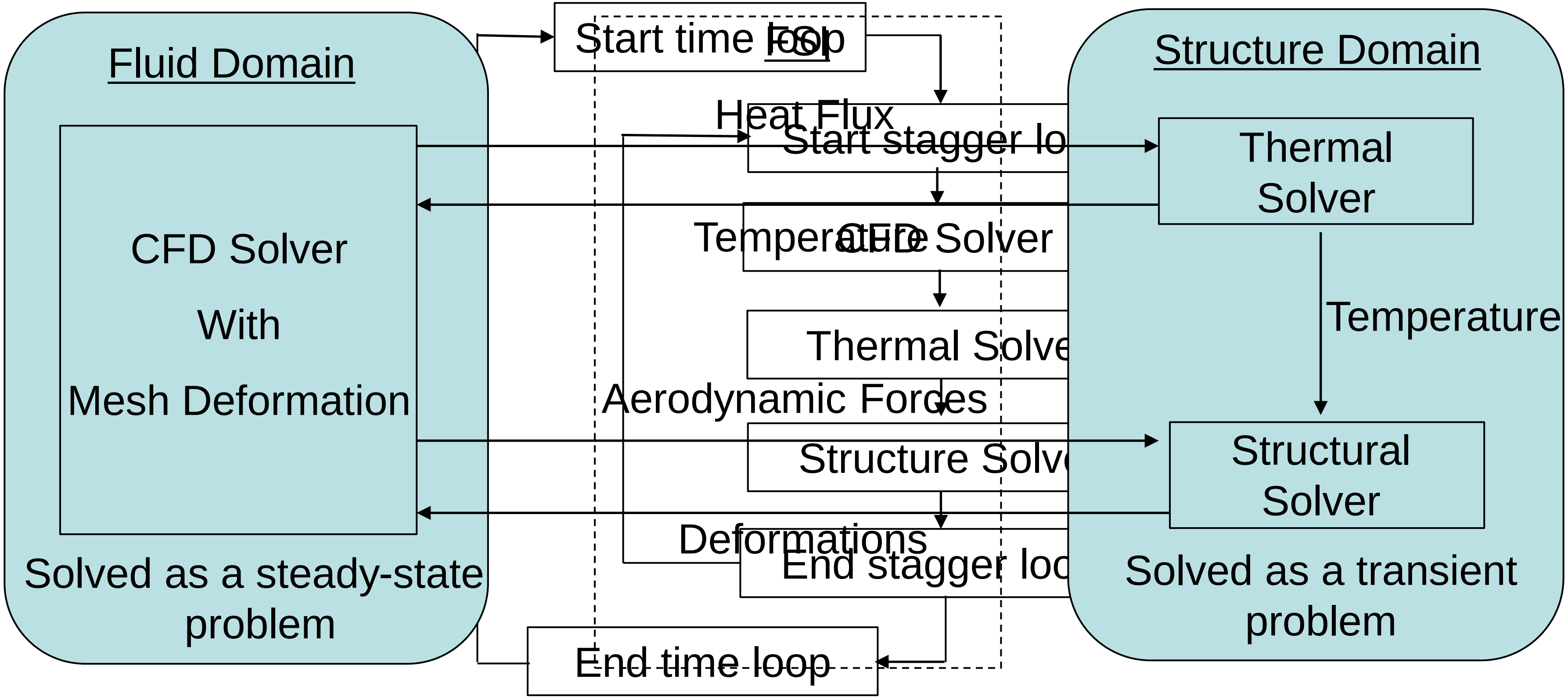
The in-house FSI :

✓ can handle **non-matching** fluid and structure **meshes** with different element types and mesh resolution.

✓ can handle **non-matching** fluid and structure **OML** geometries

*Li et al, 3D common-refinement method for non-matching meshes in partitioned variational fluid-structure analysis (2017)

AERO-THERMO-ELASTIC COUPLING



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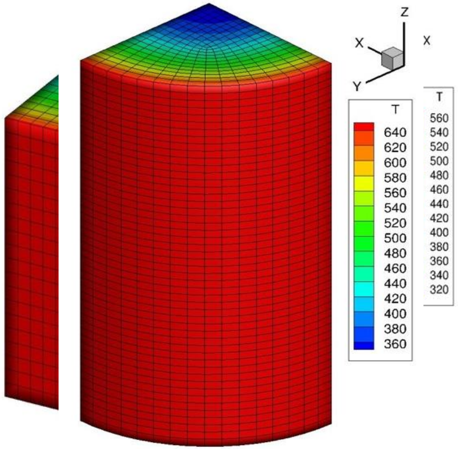
STRUCTURAL SOLVER VALIDATION
Thermal Analysis Validation

- Followed cases demonstrated in **AIAA 2019-1892** (Verification of a conjugate heat transfer tool with US3D, J.D. Reinert, A. Dwivedi, and G.V. Candler):

  - ✓ **Transient 1D heat conduction in a cube.**
    - ✓ with Dirichlet boundary conditions and constant thermal properties.
    - ✓ with Neumann boundary conditions and constant thermal properties
    - ✓ with Neumann boundary conditions and variable thermal properties.

  - ✓ **Transient 2D heat conduction on a quarter cylinder**.

  - ✓ Numerical solutions were compared against analytical solutio



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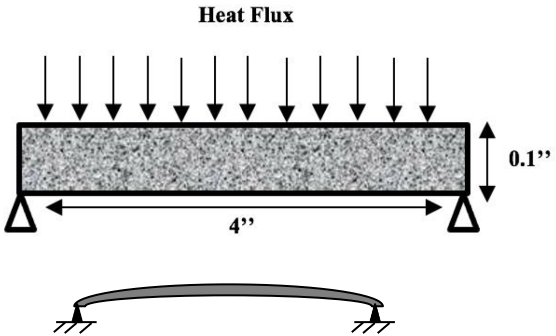
STRUCTURAL SOLVER VALIDATION
Thermo-Elastic Analysis Validation

- Thermo-elastic validation:
  - ✓ Thermo-elastic study of a heated panel case. Based on the 1988 paper by Thornton et al, titled ”Flow, Thermal, and Structural Analysis of Aerodynamically Heated Panels”
  - ✓ Numerical solutions were compared against analytical solutions.



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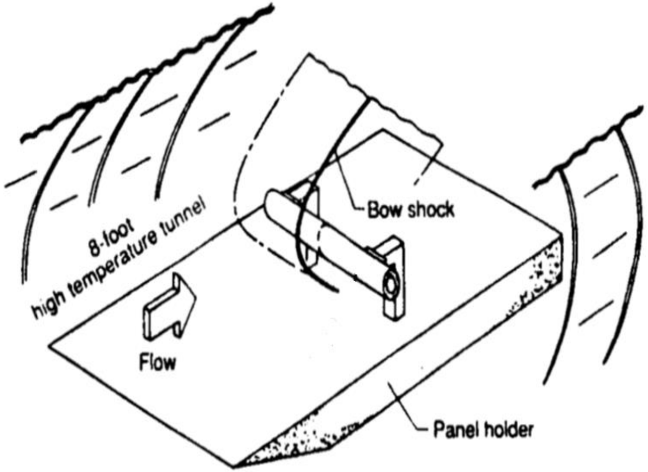
AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

- Based on the experiments conducted by Allan Wieting in the NASA Langley 8-foot High Temperature Tunnel in 1987*

Free-stream conditions	Value
Free-stream Mach number (Ma_∞)	6.47 (dimensionless)
Initial wall temperature (T_w)	294.4 K
Free-stream Reynolds number (Re_∞)	1.312×10^6 1/m
Free-stream temperature (T_∞)	241.5 K
Free-stream velocity (U_∞)	2015.43 m/s
Free-stream pressure (P_∞)	648.1 Pa



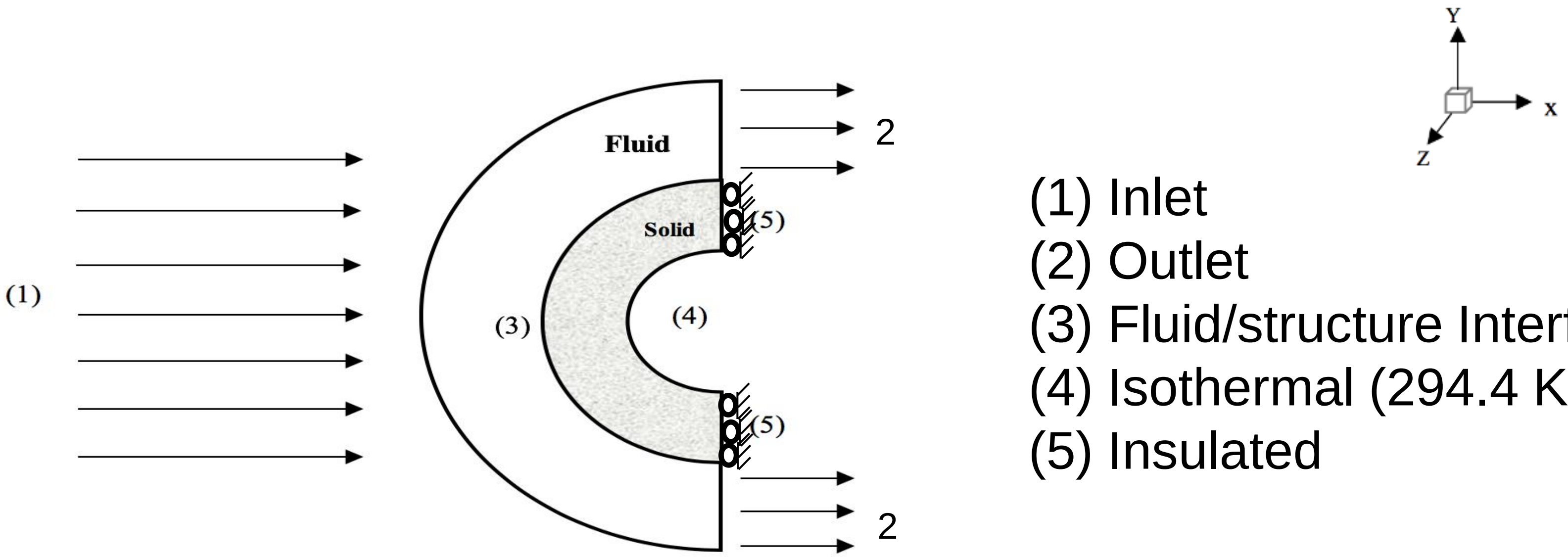
Overview of the wind tunnel experiment*

- **Description of the cylinder:**
  - **Material properties: stainless steel 321 at 400K**
  - **Dimensions: Length = 0.1143m, Diameter= 0.0762m,Thickness = 0.0127m**

*Dechaumphai et al, Fluid-Thermal-structural Study of Aerodynamically Heated Leading Edges (1988).

AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

- Summary of the applied numerical boundary conditions

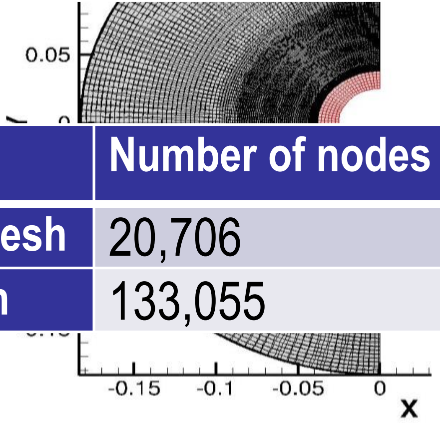


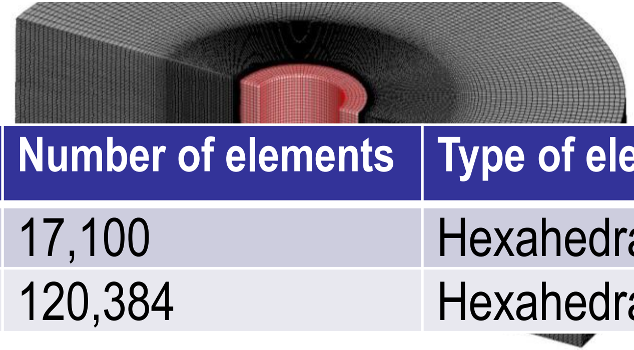
(1) Inlet
(2) Outlet
(3) Fluid/structure Interface
(4) Isothermal (294.4 K)
(5) Insulated

AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

- Description of the grids used for the numerical simulation

Fluid Mesh	Number of nodes	Number of elements	Type of elements	Wall spacing
Fluid coarse mesh	2,462,400	4,814,740	Prism	10^{-6}
Fluid fine mesh	19,763,866	39,084,360	Prism	6×10^{-7}



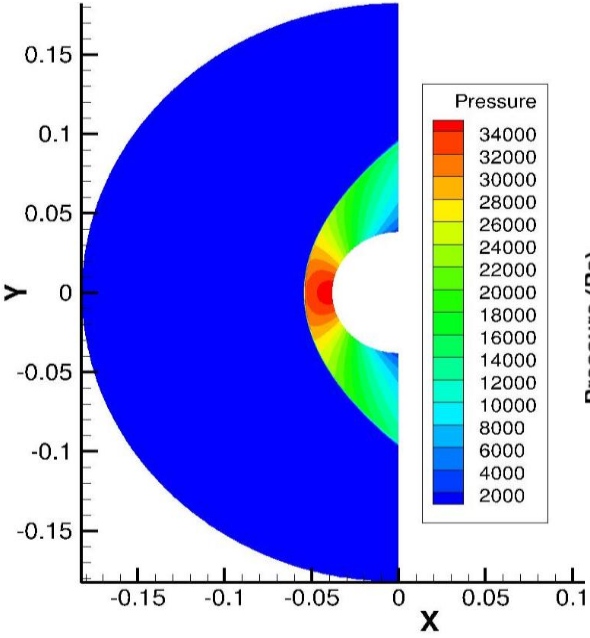


Structure Mesh	Number of nodes	Number of elements	Type of elements
Structure coarse mesh	20,706	17,100	Hexahedral
Structure fine mesh	133,055	120,384	Hexahedral

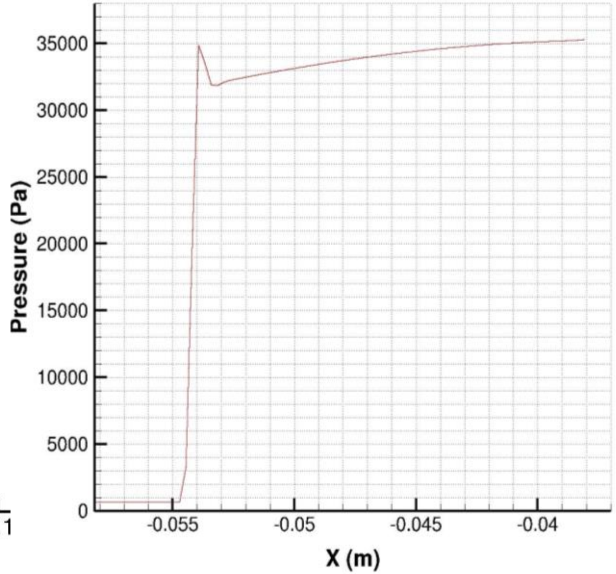
AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

- Validation of the **CFD solver** for high speed flows

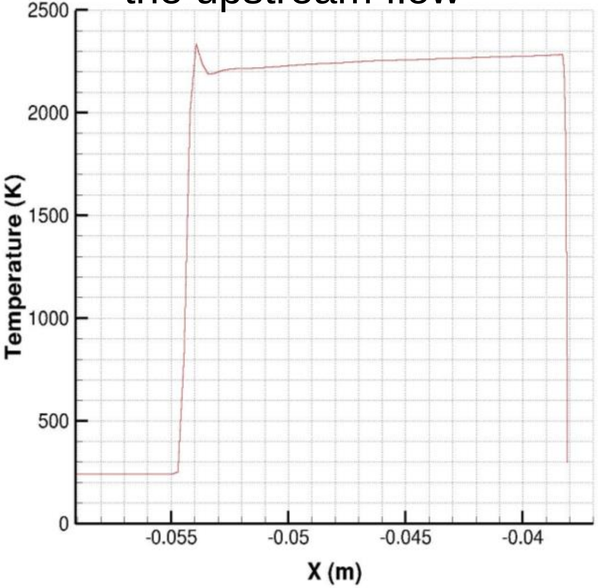
Pressure Contour



Fluid pressure
along the centerline of the upstream flow



Fluid temperature
along the centerline of the upstream flow

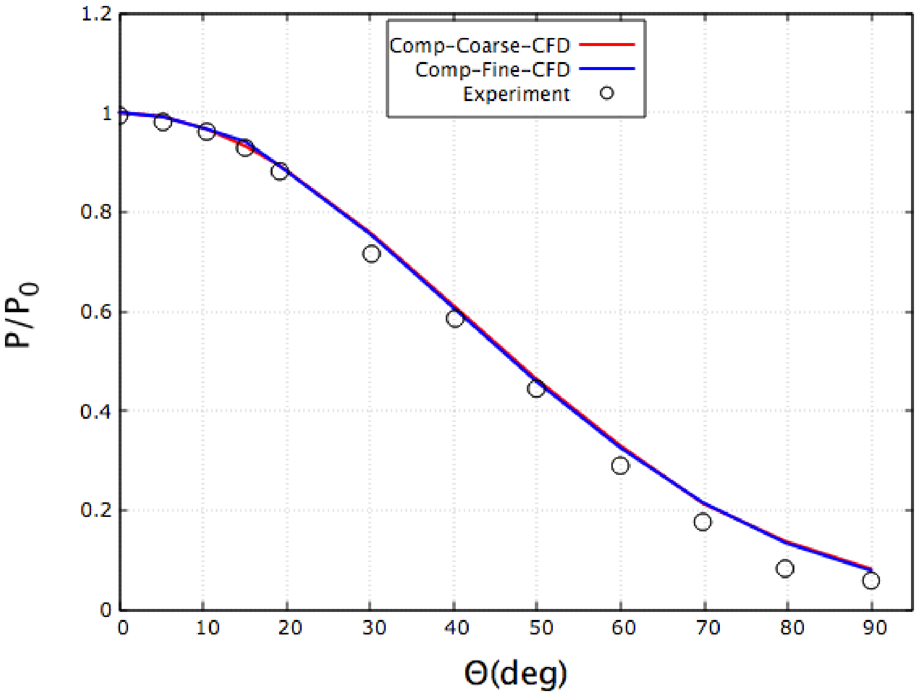


AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

➢ Validation of the **CFD solver** for high speed flows

- Stagnation point pressure values
- Comparison of computed and experimental pressures (Normalized)

Stagnation point parameters	heoretical (perfect gas)
Pressure (P_0)	5,231.16 Pa



*Wieting, A.,R, Experimental Study c

AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

➢ Validation of the **CFD solver** for high speed flows

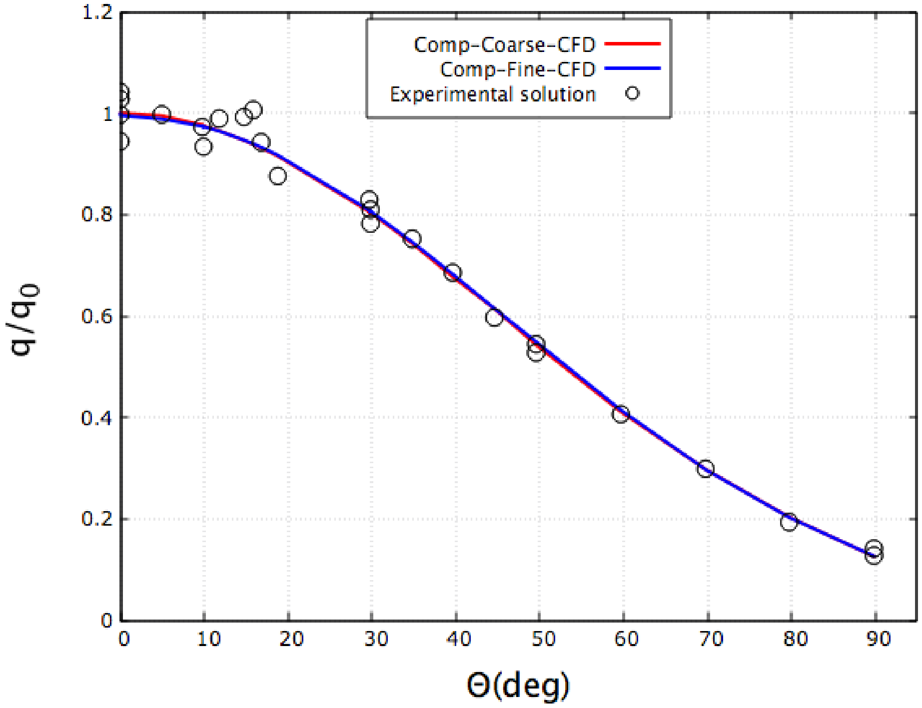
- Stagnation point heating rate values
- Comparison of the heating rate with experimental and previous work (Normalized)

Stagnation p
heating rate (

Cur

Stagnation
parameters

Heating rate



ntal*

Viscous shock-
ver solution*

0.1 KW/m²

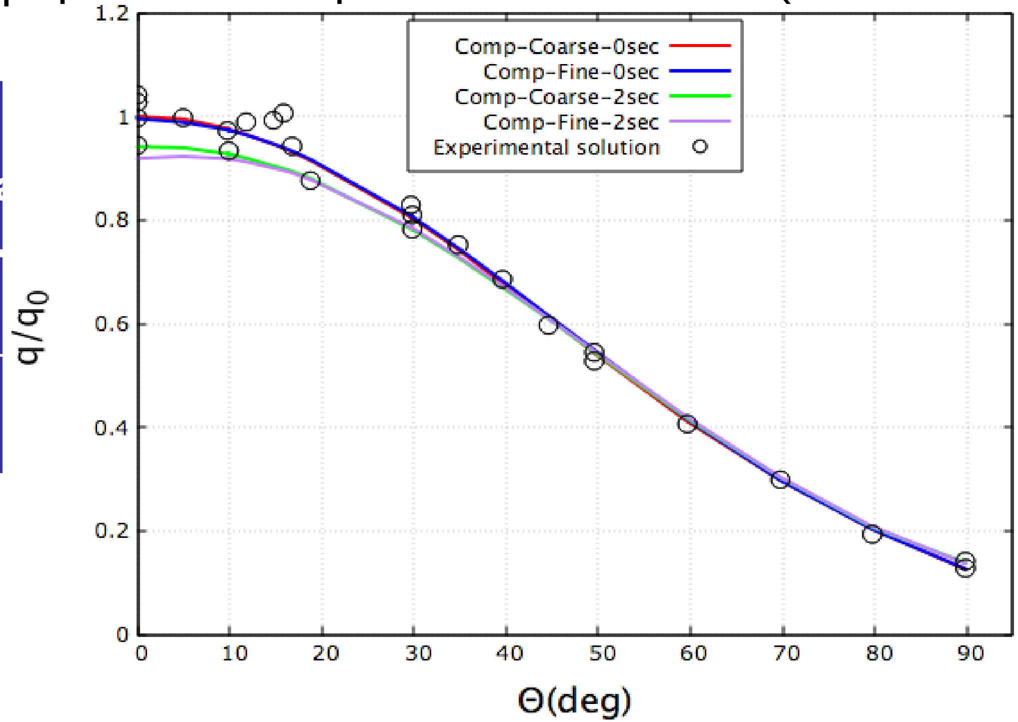
2

*Dechaumphai et al, Fluid-Thermal-structura
**Wieting, A. R., Experimental Study of Shock
**Zhang et al, Time-Adaptive, Loosely Couple

AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

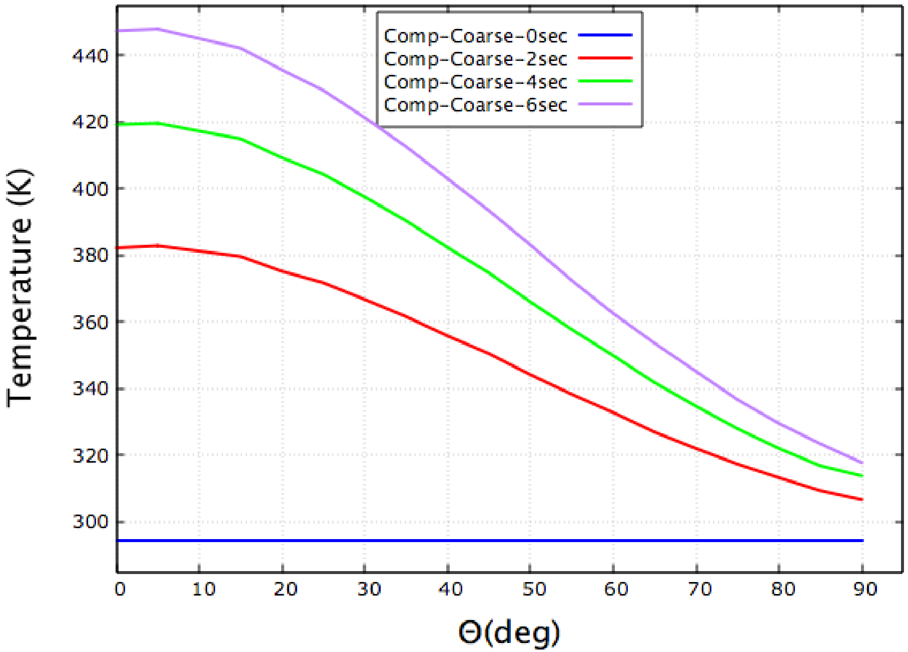
- Validation of the **coupled analysis** capability **(structural solver time step is 0.1s)**
  - Stagnation point parameters
  - Comparison of computed and experimental heat rate (Normalized)

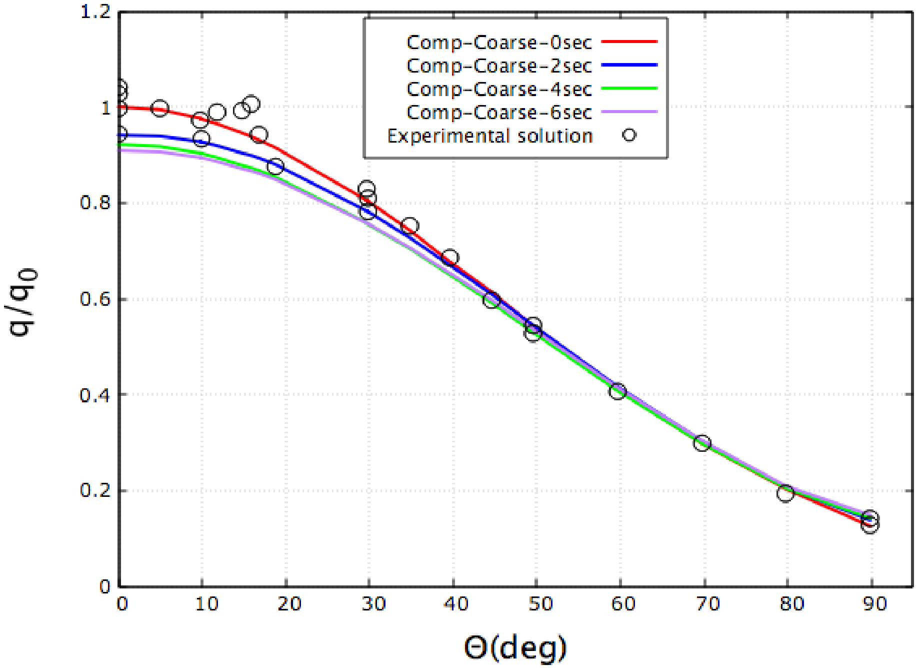
Stagnation point pa	coupled at t = 2s
	Fine mesh
Pressure (P_0)	35,147.81 Pa
Heating rate (q_0)	438.52 KW/m^2



AERO-THERMO-ELASTIC ANALYSIS RESULTS
Hypersonic Flow Over a Cylindrical Leading Edge

- Validation of the **coupled analysis** capability **(structural solver time step is 0.1s)**
  - Evolution of temperature and heat flux with time

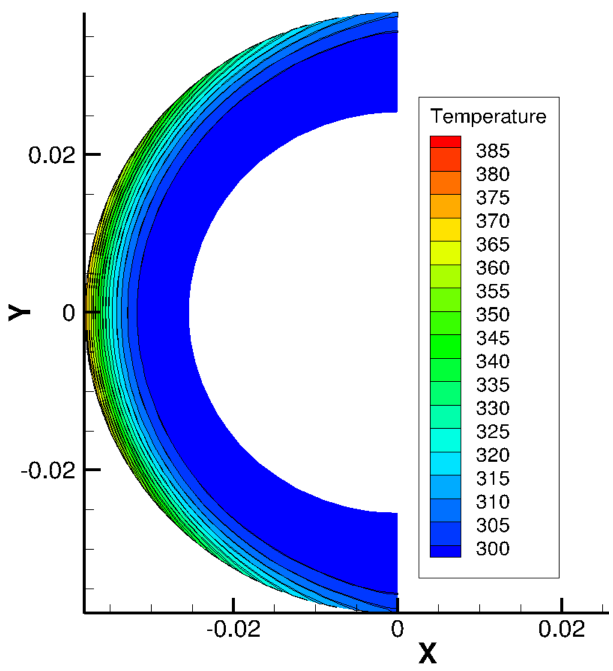




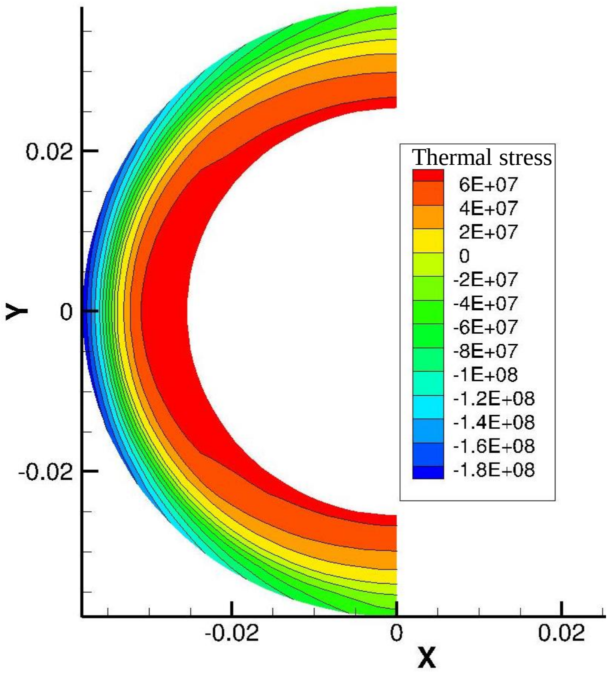
AERO-THERMO-ELASTIC ANALYSIS VALIDATION
Hypersonic Flow Over a Cylindrical Leading Edge

- Validation of the **coupled analysis** capability

Temperature(K) solution at t =2 s



Circumferential thermal stress(pa) solutions at t = 2s



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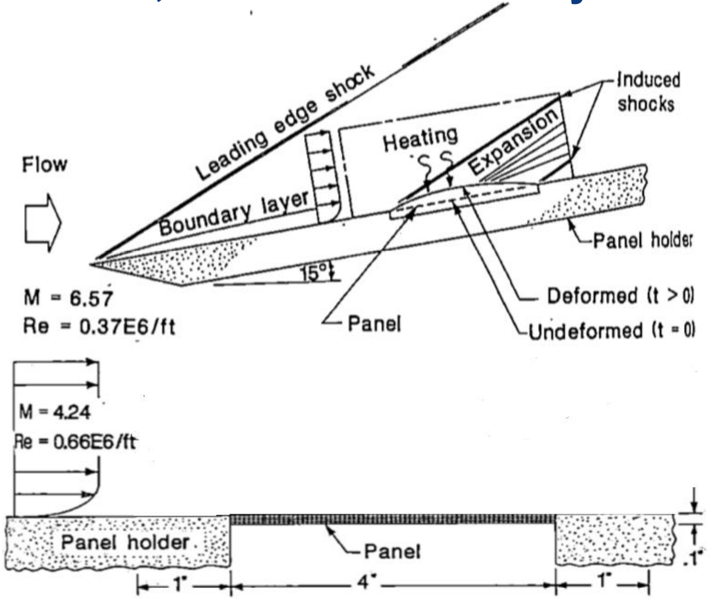
AERO-THERMO-ELASTIC ANALYSIS RESULTS
Aerodynamically Heated Panel

- Overview of the proposed wind tunnel experiment

Based on the 1988 paper by Thornton et al, titled ”Flow, Thermal, and Structural Analysis of Aerodynamically Heated Panels

Free-stream conditions	Value
Free-stream Mach number (Ma_∞)	6.57 (dimensionless)
Wall temperature (T_w)	530 K
Free-stream Reynolds number (Re_∞)	0.37×10^6 1/ft
Free-stream temperature (T_∞)	530 K
Free-stream velocity (U_∞)	6612.3 ft/s
Free-stream pressure (P_∞)	0.0971 psi

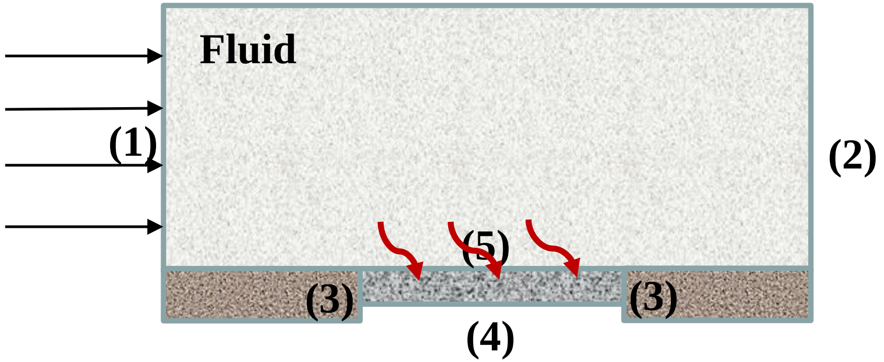


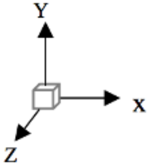
Proposed experimental set-up for the heated panel case*

*Thornton et al, Coupled Flow, Thermal, and Structural Analysis of Aerodynamically Heated Panels (1988).

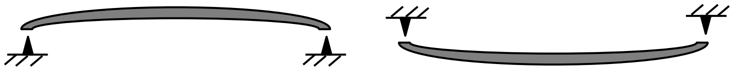
AERO-THERMO-ELASTIC ANALYSIS RESULTS
Aerodynamically Heated Panel

- Summary of the applied numerical boundary conditions





(1) Inflow
(2) Outflow
(3) Isothermal (530 R)
(4) Insulated
(5) Fluid/Structure Interface



Panel structural boundary conditions

- **Description of the panel:**
  - **Material properties: stainless steel AM-350**
  - **Dimensions: Length = 4 in, Width= 0.1 in, Thickness = 0.5 in**

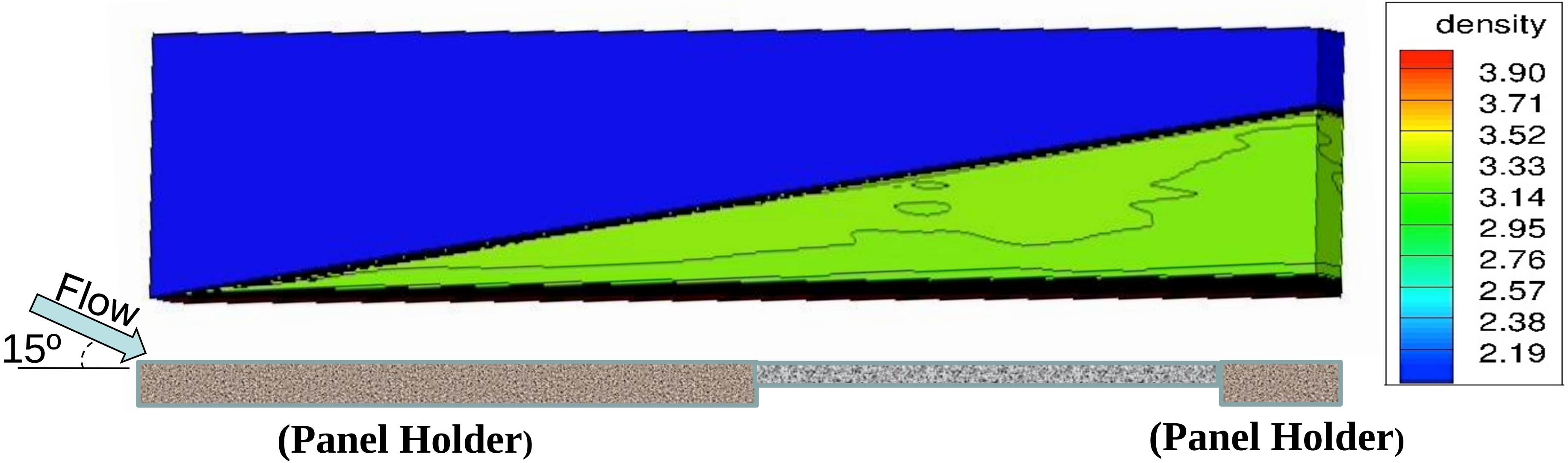
AERO-THERMO-ELASTIC ANALYSIS RESULTS
Aerodynamically Heated Panel

- Description of the grids used for the numerical simulation

Mesh	Number of nodes	Number of elements	Type of elements	Wall spacing
Fluid mesh	2,474,940	4,725,000	Prism	6×10^{-6}
Structure mesh	3,216	1,995	Hexahedral	

AERO-THERMO-ELASTIC ANALYSIS RESULTS
Aerodynamically Heated Panel

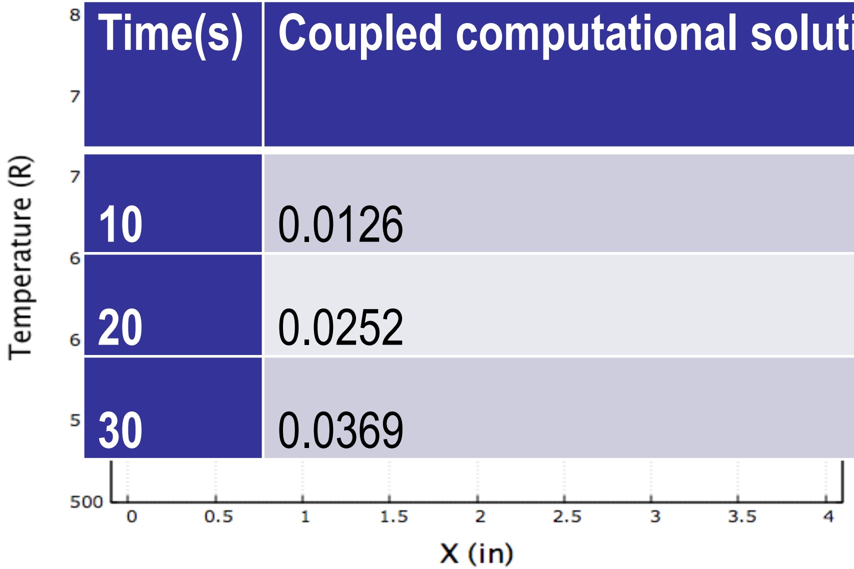
- Numerical results for the **coupled** simulation of an Aerodynamically heated panel with **convex deformation** (structural solver time step 5s)
  - Flow density distribution from t = 0s to t = 30s **(6 coupling cycles)**

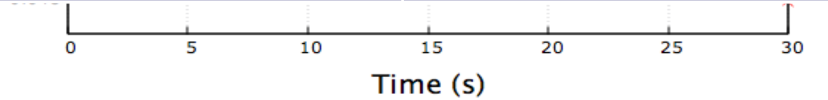


AERO-THERMO-ELASTIC ANALYSIS RESULTS
Aerodynamically Heated Panel

- Numerical results for the **coupled** simulation of an Aerodynamically heated panel with **convex deformation**

  - Panel Temperature (L/2) in Rankine
  - Evolution of panel temperature (L/2), and heat flux with time





Time(s)	Coupled computational solution	Analytical solution	Computational solutions from previous work*
10	0.0126	0.0127	0.0133
20	0.0252	0.0234	0.0239
30	0.0369	0.0336	0.0327

*Thornton et al, Coupled Flow, Thermal, and Structural Analysis of Aerodynamically Heated Panels (1988).

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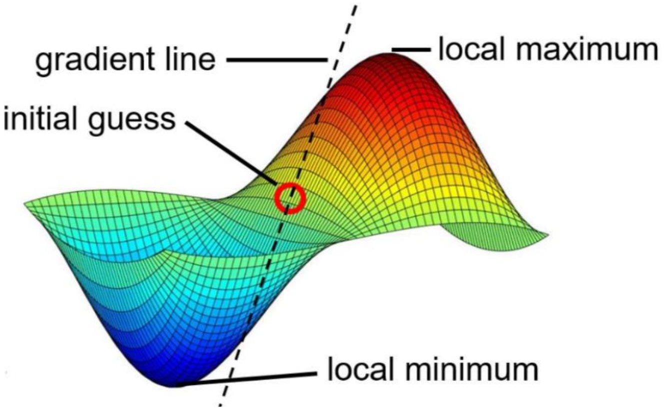
SENSITIVITY ANALYSIS & OPTIMIZATION
Gradient Based Methods

- Gradient based sensitivity derivative Analysis for optimization:
  - Finite-difference $f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \cdots \Rightarrow f'(x) = \frac{f(x+h)-f(x)}{h}$
  - Complex-step $f(x+ih) = f(x) + ihf'(x) - \frac{h^2}{2} f''(x) + \cdots \Rightarrow f'(x) = \frac{Im[f(x+ih)]}{h}$
  - Analytical (Tangent and Adjoint)
    - ✓ High Accuracy
    - ✓ Less Computationally expensive



Conceptual depiction of the gradient*

*Anderson, E., Development of an Open-Source Capability for High-Fidelity Thermoelastic Modeling and Adjoint-Based Sensitivity Analysis of Structures, PhD thesis, August 2019.

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Steady-State Tangent Formulation

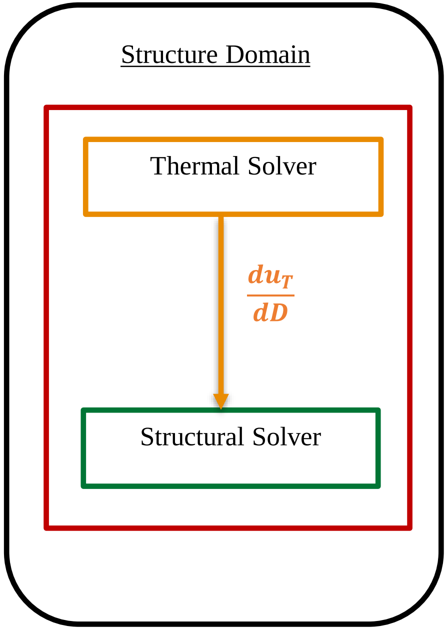
Objective function: $L = L(D, u_T(D), u_S(D))$

Sensitivities: $$\frac{dL}{dD} = \frac{\partial L}{\partial D} + \begin{bmatrix} \frac{\partial L}{\partial u_T} & \frac{\partial L}{\partial u_S} \end{bmatrix} \begin{bmatrix} \frac{\partial u_T}{\partial D} \\ \frac{\partial u_S}{\partial D} \end{bmatrix}$$

Subject to: $R_T(D, u_T, u_S) = 0$ and $R_S(D, u_T, u_S) = 0$

Constraint sensitivity eqn: $$\begin{bmatrix} \frac{\partial R_T}{\partial u_T} & \frac{\partial R_T}{\partial u_S}\ \mathbf{0} \\ \frac{\partial R_S}{\partial u_T} & \frac{\partial R_S}{\partial u_S} \end{bmatrix} \begin{bmatrix} \frac{\partial u_T}{\partial D} \\ \frac{\partial u_S}{\partial D} \end{bmatrix} = \begin{bmatrix} -\frac{\partial R_T}{\partial D} \\ -\frac{\partial R_S}{\partial D} \end{bmatrix}$$

Final Form: $$\frac{dL}{dD} = \frac{\partial L}{\partial D} + \begin{bmatrix} \frac{\partial L}{\partial u_T} & \frac{\partial L}{\partial u_S} \end{bmatrix} \begin{bmatrix} \frac{\partial R_T}{\partial u_T} & 0 \\ \frac{\partial R_S}{\partial u_T} & \frac{\partial R_S}{\partial u_S} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\partial R_T}{\partial D} \\ -\frac{\partial R_S}{\partial D} \end{bmatrix}$$



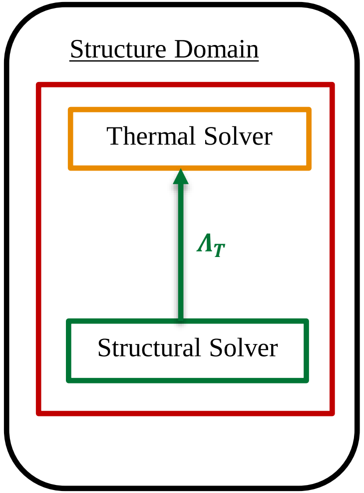
✗ For multiple **D,** multiple linear solutions required

THERMO-ELASTIC SENSITIVITY FORMULATION
Steady-State Adjoint Formulation

Adjoint sensitivities: $\frac{dL}{dD}^T = \frac{\partial L}{\partial D}^T + \begin{bmatrix} \frac{\partial u_T}{\partial D}^T & \frac{\partial u_S}{\partial D}^T \end{bmatrix} \begin{bmatrix} \frac{\partial L}{\partial u_T}^T \\ \frac{\partial L}{\partial u_S}^T \end{bmatrix}$

Disciplinary adjoints: $\begin{bmatrix} \Lambda_T \\ \Lambda_S \end{bmatrix} = \begin{bmatrix} \frac{\partial R_T}{\partial u_T}^T & \frac{\partial R_S}{\partial u_T}^T \\ \frac{\partial R_T}{\partial u_S}^T & \frac{\partial R_S}{\partial u_S}^T \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial u_T}^T \\ \frac{\partial L}{\partial u_S}^T \end{bmatrix}$

Linear Adjoint System: $\begin{bmatrix} \frac{\partial R_T}{\partial u_T}^T & \frac{\partial R_S}{\partial u_T}^T \\ \frac{\partial R_T}{\partial u_S}^T \; (0) & \frac{\partial R_S}{\partial u_S}^T \end{bmatrix} \begin{bmatrix} \Lambda_T \\ \Lambda_S \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial u_T}^T \\ \frac{\partial L}{\partial u_S}^T \end{bmatrix}$



Final Form: $\frac{dL}{dD} = \frac{\partial L}{\partial D} + \begin{bmatrix} \Lambda_T^T & \Lambda_S^T \end{bmatrix} \begin{bmatrix} -\frac{\partial R_T}{\partial D} \\ -\frac{\partial R_S}{\partial D} \end{bmatrix}$

- ✓ No dependence on **D** during linear solution
- ✓ Effect of **D** confined to final matrix-vector product

THERMO-ELASTIC SENSITIVITY FORMULATION
Transient Tangent Formulation

$$\frac{\partial R}{\partial u} = \begin{bmatrix} \left[\frac{\partial R^0}{\partial u^0}\right] & 0 & 0 & 0 & \dots \\ \left[\frac{\partial R^1}{\partial u^0}\right] & \left[\frac{\partial R^1}{\partial u^1}\right] & 0 & 0 & \dots \\ 0 & \left[\frac{\partial R^2}{\partial u^1}\right] & \left[\frac{\partial R^2}{\partial u^2}\right] & 0 & \dots \\ 0 & 0 & \left[\frac{\partial R^3}{\partial u^2}\right] & \left[\frac{\partial R^3}{\partial u^3}\right] & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Temporal domain with two time-steps n and n-1:

$$\begin{bmatrix} \frac{\partial R_T}{\partial u_T} & \frac{\partial R_T}{\partial u_S} \\ \frac{\partial R_S}{\partial u_T} & \frac{\partial R_S}{\partial u_S} \end{bmatrix} \begin{bmatrix} \frac{\partial u_T}{\partial D} \\ \frac{\partial u_S}{\partial D} \end{bmatrix} = \begin{bmatrix} -\frac{\partial R_T}{\partial D} \\ -\frac{\partial R_S}{\partial D} \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} \left[\frac{\partial {R_T}^{n-1}}{\partial {u_T}^{n-1}}\right] & 0 & \left[\frac{\partial {R_T}^{n-1}}{\partial {u_S}^{n-1}}\right] & 0 \\ \left[\frac{\partial {R_T}^{n}}{\partial {u_T}^{n-1}}\right] & \left[\frac{\partial {R_T}^{n}}{\partial {u_T}^{n}}\right] & \left[\frac{\partial {R_T}^{n}}{\partial {u_S}^{n-1}}\right] & \left[\frac{\partial {R_T}^{n}}{\partial {u_S}^{n}}\right] \\ \hline \left[\frac{\partial {R_S}^{n-1}}{\partial {u_T}^{n-1}}\right] & 0 & \left[\frac{\partial {R_S}^{n-1}}{\partial {u_S}^{n-1}}\right] & 0 \\ \left[\frac{\partial {R_S}^{n}}{\partial {u_T}^{n-1}}\right] & \left[\frac{\partial {R_S}^{n}}{\partial {u_T}^{n}}\right] & \left[\frac{\partial {R_S}^{n}}{\partial {u_S}^{n-1}}\right] & \left[\frac{\partial {R_S}^{n}}{\partial {u_S}^{n}}\right] \end{array}\right] \begin{bmatrix} \frac{\partial {u_T}^{n-1}}{\partial D} \\ \frac{\partial {u_T}^{n}}{\partial D} \\ \hline \frac{\partial {u_S}^{n-1}}{\partial D} \\ \frac{\partial {u_S}^{n}}{\partial D} \end{bmatrix} = \begin{bmatrix} -\frac{\partial {R_T}^{n-1}}{\partial D} \\ -\frac{\partial {R_T}^{n}}{\partial D} \\ \hline -\frac{\partial {R_S}^{n-1}}{\partial D} \\ -\frac{\partial {R_S}^{n}}{\partial D} \end{bmatrix}$$

THERMO-ELASTIC SENSITIVITY FORMULATION
Transient Adjoint Analysis Formulation

$$\left[\frac{\partial R}{\partial u}\right]^T = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \left[\frac{\partial R^{n-3}}{\partial u^{n-3}}\right]^T & \left[\frac{\partial R^{n-2}}{\partial u^{n-3}}\right]^T & 0 & 0 \\ \cdots & 0 & \left[\frac{\partial R^{n-2}}{\partial u^{n-2}}\right]^T & \left[\frac{\partial R^{n-1}}{\partial u^{n-2}}\right]^T & 0 \\ \cdots & 0 & 0 & \left[\frac{\partial R^{n-1}}{\partial u^{n-1}}\right]^T & \left[\frac{\partial R^{n}}{\partial u^{n-1}}\right]^T \\ \cdots & 0 & 0 & 0 & \left[\frac{\partial R^{n}}{\partial u^{n}}\right]^T \end{bmatrix}$$

Temporal domain with two time-steps n and n-1:

$$\begin{bmatrix} \frac{\partial R_T}{\partial u_T}^T & \frac{\partial R_S}{\partial u_T}^T \\ \frac{\partial R_T}{\partial u_S}^T & \frac{\partial R_S}{\partial u_S}^T \end{bmatrix} \begin{bmatrix} \Lambda_T \\ \Lambda_S \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial u_T}^T \\ \frac{\partial L}{\partial u_S}^T \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} \left[\frac{\partial R_T{}^{n-1}}{\partial u_T{}^{n-1}}\right]^T & \left[\frac{\partial R_S{}^{n-1}}{\partial u_T{}^{n-1}}\right]^T & \left[\frac{\partial R_T{}^{n}}{\partial u_T{}^{n-1}}\right]^T & \left[\frac{\partial R_S{}^{n}}{\partial u_T{}^{n-1}}\right]^T \\ 0 & \left[\frac{\partial R_S{}^{n-1}}{\partial u_S{}^{n-1}}\right]^T & 0 & \left[\frac{\partial R_S{}^{n}}{\partial u_S{}^{n-1}}\right]^T \\ \hline 0 & 0 & \left[\frac{\partial R_T{}^{n}}{\partial u_T{}^{n}}\right]^T & \left[\frac{\partial R_S{}^{n}}{\partial u_T{}^{n}}\right]^T \\ 0 & 0 & 0 & \left[\frac{\partial R_S{}^{n}}{\partial u_S{}^{n}}\right]^T \end{array}\right] \begin{bmatrix} \Lambda_T^{n-1} \\ \Lambda_S^{n-1} \\ \hline \Lambda_T^{n} \\ \Lambda_S^{n} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial u_T^{n-1}}^T \\ \frac{\partial L}{\partial u_S^{n-1}}^T \\ \hline \frac{\partial L}{\partial u_T^{n}}^T \\ \frac{\partial L}{\partial u_S^{n}}^T \end{bmatrix}$$

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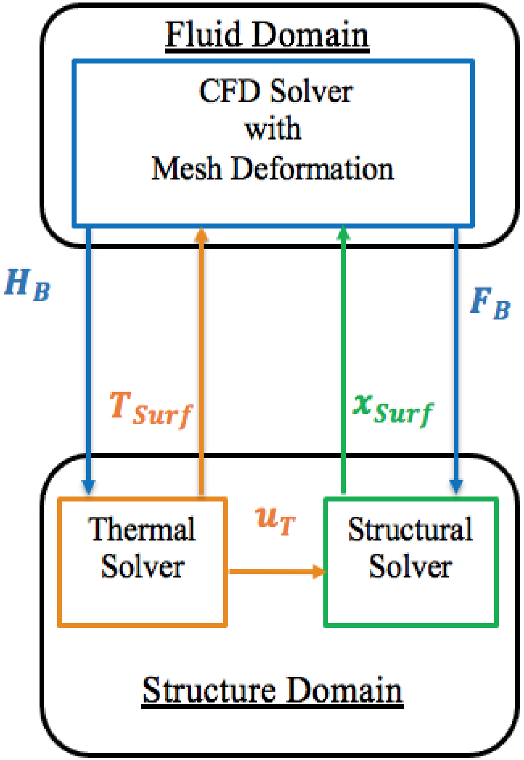
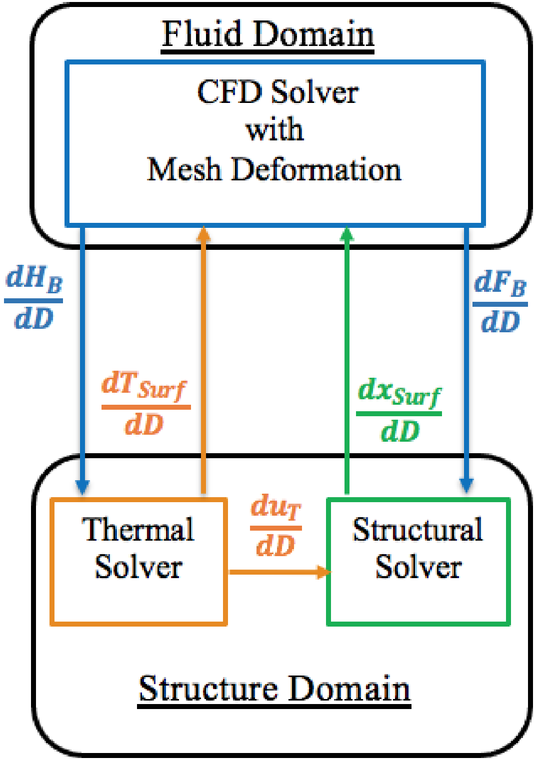
AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION

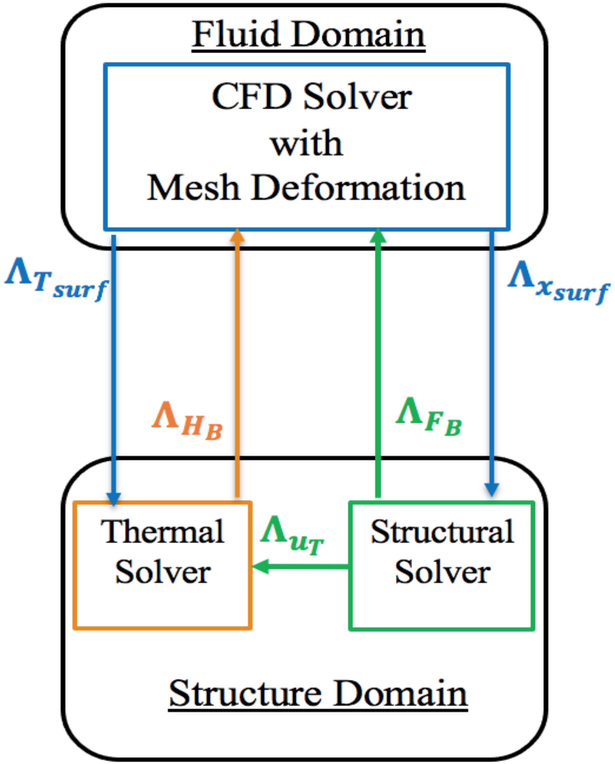
Analysis Procedure



Tangent Procedure



Adjoint Procedure



AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION
Tangent Formulation

Objective function: $L = L(D, u_x(D), u_F(D), u_T(D), u_s(D))$

Variable definitions:

- $\boldsymbol{u_x}$: CFD grid point coordinates
- $\boldsymbol{u_F}$: CFD flow values
- $\boldsymbol{u_T}$: Structural temperature values
- $\boldsymbol{u_S}$: Structural displacements
- ***D*** : Design variables

Sensitivities:

$$\frac{dL}{dD} = \frac{\partial L}{\partial D} + \begin{bmatrix} \frac{\partial L}{\partial u_x} & \frac{\partial L}{\partial u_F} & \frac{\partial L}{\partial u_T} & \frac{\partial L}{\partial u_S} \end{bmatrix} \begin{bmatrix} \frac{\partial u_x}{\partial D} \\ \frac{\partial u_F}{\partial D} \\ \frac{\partial u_T}{\partial D} \\ \frac{\partial u_S}{\partial D} \end{bmatrix}$$

AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION
Tangent Formulation

Constraints (Residual) Equations:

CFD *Mesh Deformation Equations*: $\boldsymbol{R_x\left(u_x(D), x_{surf}(D), D\right) \ = 0}$

Flow Equations: $\boldsymbol{R_F\left(u_F(D), T_{surf}(D), u_x(D), D\right) \ = 0}$

FSI transfer of forces: $\boldsymbol{G_S(F_B(u_F(D), u_x(D))) = 0}$

FSI transfer of heat fluxes: $\boldsymbol{G_T(H_B(u_F(D), u_x(D))) = 0}$

Structural displacement equations: $\boldsymbol{R_S(u_S, F_B(u_F(D), u_x(D)), D) = 0}$

Structural temperature equations: $\boldsymbol{R_T(u_T, H_B(u_F(D), u_x(D)), D) = 0}$

FSI transfer of displacements: $\boldsymbol{G'_S\left(x_{surf}(D), u_S(D)\right) = 0}$

FSI transfer of temperatures: $\boldsymbol{G'_T\left(T_{surf}(D), u_T(D)\right) = 0}$

AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION
Tangent Formulation

Linearized constraints gives tangent sensitivities:

$$\begin{bmatrix}
\frac{\partial R_x}{\partial u_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial R_x}{\partial x_{surf}} & 0 \\
\frac{\partial R_F}{\partial u_x} & \frac{\partial R_F}{\partial u_F} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial R_F}{\partial T_{surf}} \\
-\frac{\partial F_B}{\partial u_x} & -\frac{\partial F_B}{\partial u_f} & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\partial H_B}{\partial u_x} & -\frac{\partial H_B}{\partial u_F} & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\partial G_S}{\partial F_B} & 0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\partial G_T}{\partial H_B} & 0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\partial R_S}{\partial G_S} & 0 & \frac{\partial R_S}{\partial u_S} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\partial R_T}{\partial G_T} & 0 & \frac{\partial R_T}{\partial u_T} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial G'_S}{\partial u_S} & 0 & \frac{\partial G'_S}{\partial x_{surf}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial G'_T}{\partial u_T} & 0 & \frac{\partial G'_T}{\partial T_{surf}}
\end{bmatrix}
\begin{bmatrix}
\frac{du_x}{dD} \\ \frac{du_F}{dD} \\ \frac{dF_B}{dD} \\ \frac{dH_B}{dD} \\ \frac{dG_S}{dD} \\ \frac{dG_T}{dD} \\ \frac{du_S}{dD} \\ \frac{du_T}{dD} \\ \frac{dx_{surf}}{dD} \\ \frac{dT_{surf}}{dD}
\end{bmatrix}
=
\begin{bmatrix}
-\frac{\partial R_x}{\partial D} \\ -\frac{\partial R_F}{\partial D} \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{\partial R_S}{\partial D} \\ -\frac{\partial R_T}{\partial D} \\ 0 \\ 0
\end{bmatrix}$$

AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION
Adjoint Formulation

Transpose gives adjoint equations:

$$
\begin{bmatrix}
\frac{\partial R_x}{\partial u_x}^T & \frac{\partial R_F}{\partial u_x}^T & -\frac{\partial F_B}{\partial u_x}^T & -\frac{\partial H_B}{\partial u_x}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\partial R_F}{\partial u_F}^T & -\frac{\partial F_B}{\partial u_F}^T & -\frac{\partial H_B}{\partial u_F}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I & 0 & \frac{\partial G_S}{\partial F_B}^T & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & \frac{\partial G_T}{\partial H_B}^T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I & 0 & \frac{\partial R_S}{\partial G_T}^T & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I & 0 & \frac{\partial R_T}{\partial G_T}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial R_S}{\partial u_S}^T & 0 & \frac{\partial G'_S}{\partial u_S}^T & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial R_T}{\partial u_T}^T & 0 & \frac{\partial G'_T}{\partial u_T}^T \\
\frac{\partial R_x}{\partial x_{surf}}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial G'_S}{\partial x_{surf}}^T & 0 \\
0 & \frac{\partial R_F}{\partial T_{surf}}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial G'_T}{\partial T_{surf}}^T
\end{bmatrix}
\begin{bmatrix}
\Lambda_{u_x} \\ \Lambda_{u_F} \\ \Lambda_{F_B} \\ \Lambda_{H_B} \\ \Lambda_{G_S} \\ \Lambda_{G_T} \\ \Lambda_{u_s} \\ \Lambda_{u_T} \\ \Lambda_{x_{surf}} \\ \Lambda_{T_{surf}}
\end{bmatrix}
=
\begin{bmatrix}
\frac{\partial L}{\partial u_x}^T \\ \frac{\partial L}{\partial u_F}^T \\ 0 \\ 0 \\ 0 \\ \frac{\partial L}{\partial u_s}^T \\ \frac{\partial L}{\partial u_T}^T \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

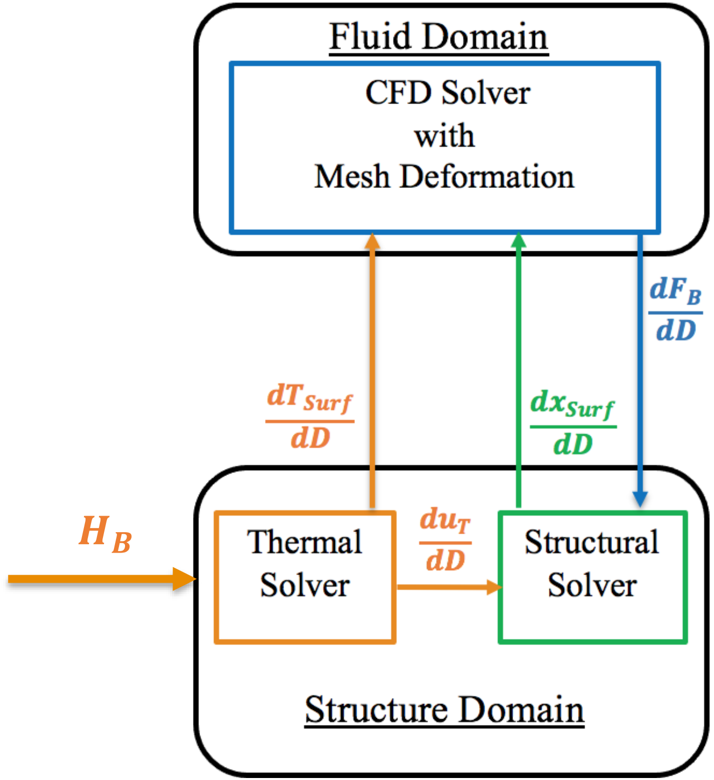
AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION
Adjoint Formulation

$$
\begin{bmatrix}
\frac{\partial R_x}{\partial u_x}^T & \frac{\partial R_F}{\partial u_x}^T & -\frac{\partial F_B}{\partial u_x}^T & -\frac{\partial H_B}{\partial u_x}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\partial R_F}{\partial u_F}^T & -\frac{\partial F_B}{\partial u_F}^T & -\frac{\partial H_B}{\partial u_F}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I & 0 & \frac{\partial G_S}{\partial F_B}^T & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & \frac{\partial G_T}{\partial H_B}^T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I & 0 & \frac{\partial R_S}{\partial G_T}^T & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I & 0 & \frac{\partial R_T}{\partial G_T}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial R_S}{\partial u_S}^T & 0 & \frac{\partial G'_S}{\partial u_S}^T & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial R_T}{\partial u_T}^T & 0 & \frac{\partial G'_T}{\partial u_T}^T \\
\frac{\partial R_x}{\partial x_{surf}}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial G'_S}{\partial x_{surf}}^T & 0 \\
0 & \frac{\partial R_F}{\partial T_{surf}}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial G'_T}{\partial T_{surf}}^T
\end{bmatrix}
\begin{bmatrix}
\Lambda_{u_x} \\ \Lambda_{u_F} \\ \Lambda_{F_B} \\ \Lambda_{H_B} \\ \Lambda_{G_S} \\ \Lambda_{G_T} \\ \Lambda_{u_s} \\ \Lambda_{u_T} \\ \Lambda_{x_{surf}} \\ \Lambda_{T_{surf}}
\end{bmatrix}
=
\begin{bmatrix}
\frac{\partial L}{\partial u_x}^T \\ \frac{\partial L}{\partial u_F}^T \\ 0 \\ 0 \\ 0 \\ \frac{\partial L}{\partial u_s}^T \\ \frac{\partial L}{\partial u_T}^T \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

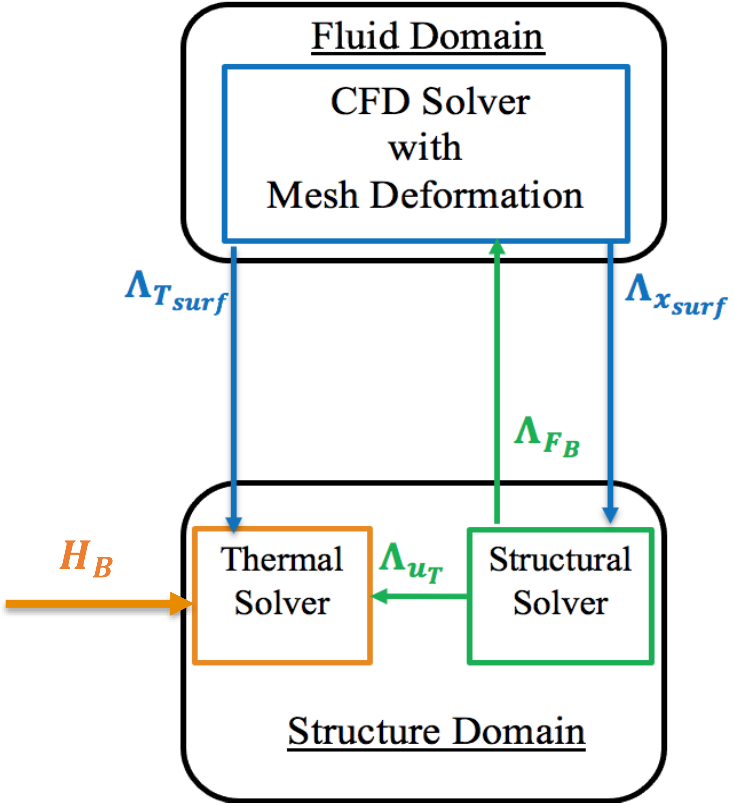
$$
\frac{dL}{dD}^T = \frac{\partial L}{\partial D}^T + \left[-\frac{\partial R_x}{\partial D}^T \quad -\frac{\partial R_F}{\partial D}^T \quad -\frac{\partial R_T}{\partial D}^T \quad -\frac{\partial R_S}{\partial D}^T \right] \begin{bmatrix} \Lambda_{u_x} \\ \Lambda_{u_f} \\ \Lambda_{u_T} \\ \Lambda_{u_S} \end{bmatrix}
$$

AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION

Tangent Procedure



Adjoint Procedure



AERO-THERMO-ELASTIC SENSITIVITY IMPLEMENTATION

- Objectives (and constraints) may be formulated based on:
  - ✓ Functional of flow quantities (C_D, C_L, C_m, etc.)
  - ✓ Functional of structural quantities (Modulus E, thickness, density,…)
  - ✓ Functional of thermal quantities (conductivity k, thermal expansion, …)
  - ✓ Combinations of above in weighted penalty formulation

- Design Variable types
  - ✓ Based on material (structural) properties
  - ✓ Based on OML shape parameters
  - ✓ Based on flow parameters (Mach, angle of attack, etc.)

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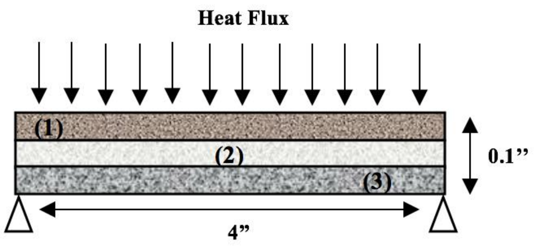
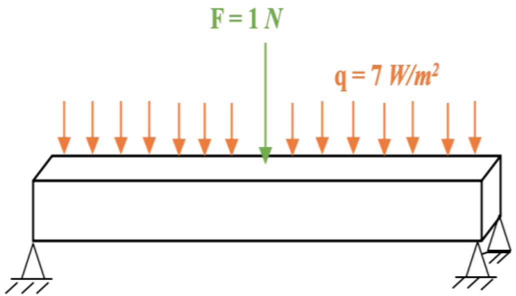
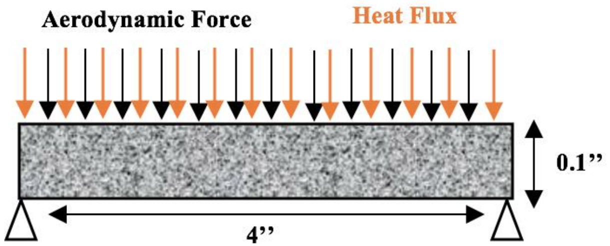
THERMO-ELASTIC SENSITIVITY ANALYSIS & OPTIMIZATION RESULTS

- ✓ Steady-state thermal optimization for a Multi-Material panel with applied heat flux.
- ✓ Steady-state thermo-elastic optimization for a Rectangular bar with applied force.
- ✓ Steady-state thermo-elastic optimization with large number of design variables.
- ✓ Transient thermo-elastic optimization on a Panel with applied heat flux and aerodynamic forces







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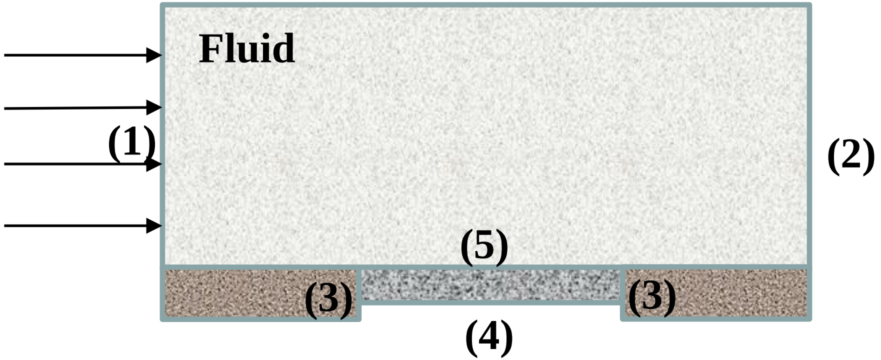
Aero-Thermo-Elastic Sensitivity Verification

- **Computational set-up:**



(1) Inflow
(2) Outflow
(3) Isothermal (530 R)
(4) Insulated
(5) Fluid/Structure Interface

- **Description of the panel:**

  - **Material properties: stainless steel AM-350**
  - **Dimensions: Length = 4 in, Width= 0.1 in, Thickness = 0.5 in**

Mesh	Number of nodes	Number of elements	Type of elements	Wall spacing
Fluid mesh	2,474,940	4,725,000	Prism	6×10^{-6}
Structure mesh	3,216	1,995	Hexahedral	

AERO-THERMO-ELASTIC SENSITIVITY ANALYSIS & OPTIMIZATION RESULTS
Aero-Thermo-Elastic Sensitivity Verification

- **Objective function:** $L = (C_x)^2$

- **Design variables:** $$\begin{cases} k = k_0 + 10^{-4} D_1 \\ Thickness = Thickness_{intial} \times D_2 \end{cases}$$

- **Sensitivity verification:** $\boldsymbol{D_1}$

Time step	Adjoint	Tangent	Finite-Difference
1	-6.96927382827**343**$\times 10^{-6}$	-6.96927382827**098**$\times 10^{-6}$	-7.3292648685$\times 10^{-6}$
2	-2.09733547485**967**$\times 10^{-5}$	-2.09733547486**444**$\times 10^{-5}$	-2.2907747595$\times 10^{-5}$
3	-	-4.19421030209**051**$\times 10^{-5}$	-4.6948589964$\times 10^{-5}$

AERO-THERMO-ELASTIC SENSITIVITY ANALYSIS & OPTIMIZATION RESULTS
Aero-Thermo-Elastic Optimization

- **Objective function:** $L(t) = (C_x(t_{final}))^2 + \omega_1(k - k_0)^2 + \omega_2(Mass - Mass_0)^2$

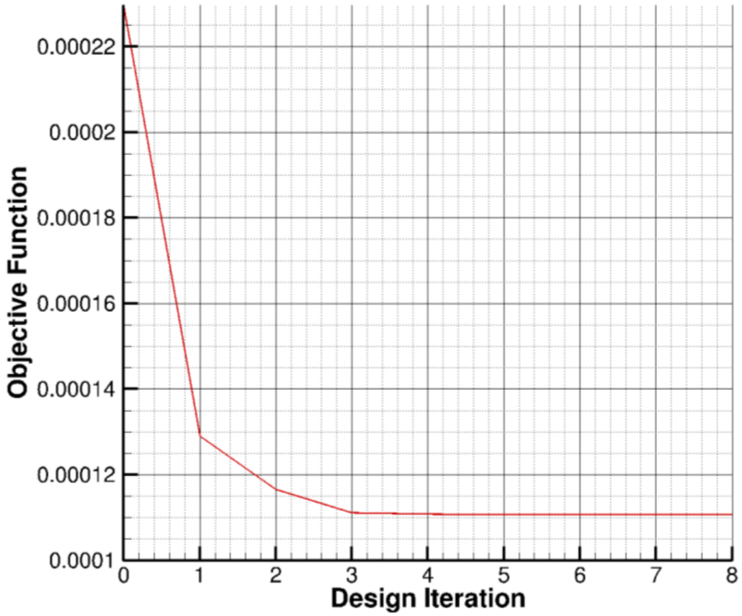
- **Design variables:** $\begin{cases} k = k_0 + 10^{-4} D_1 \\ Thickness = Thickness_{intial} \times D_2 \end{cases}$

- Initial and optimized material properties **(using Δt = 1 sec)**

Material Properties	Initial	Optimized 1 coupled time step	Optimized 5 coupled time steps
Thermal Conductivity (k)	0.00012864 *BTU/(s.in.R)*	0.0002059 *BTU/(s.in.R)*	0.000214203 BTU/(s.in.R)
Thickness	*1 in*	*2.504113 in*	2.5134456 *in*

Aero-Thermo-Elastic Optimization

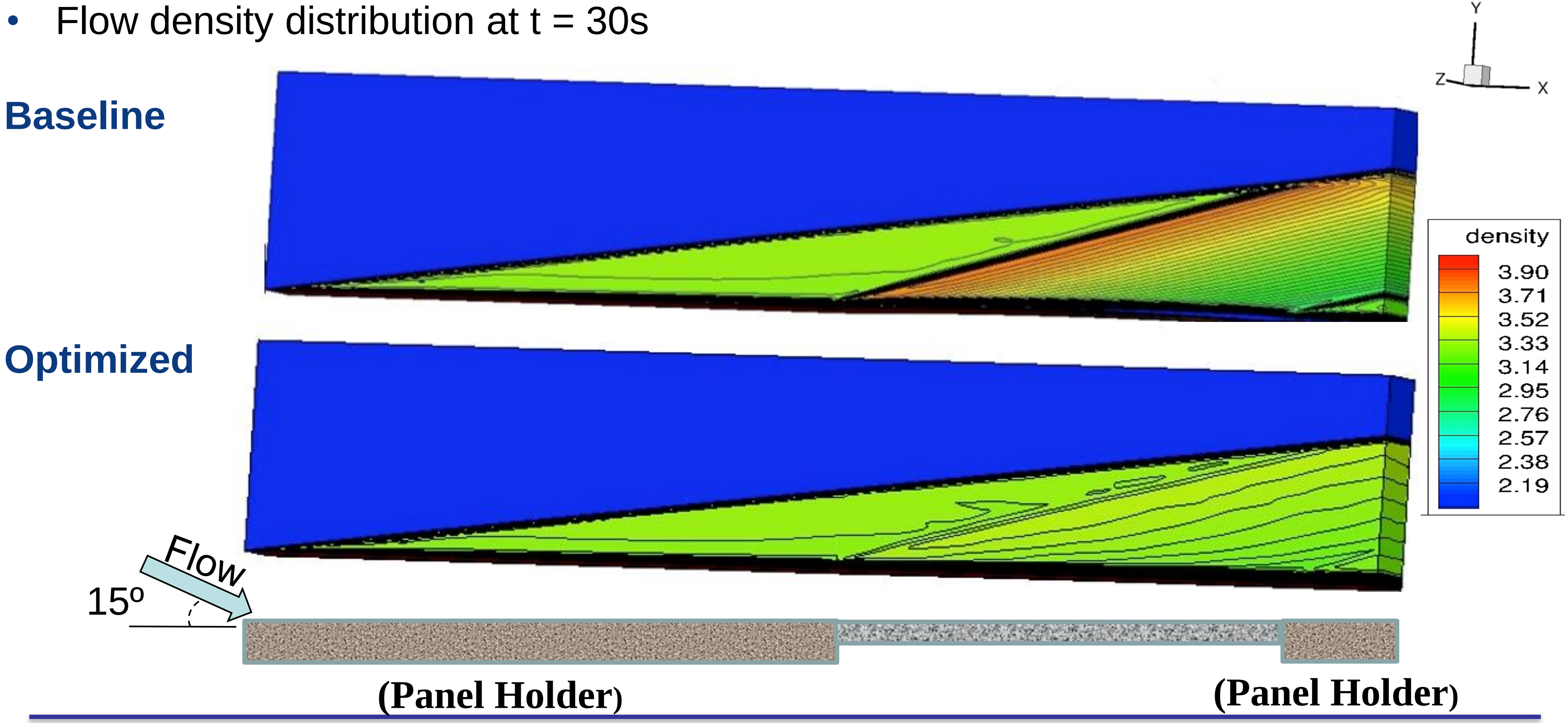
- Convergence of the aero-thermo-elastic optimization process



5 coupled time steps

Aero-Thermo-Elastic Optimization

- Flow density distribution at t = 30s



OUTLINE

- Introduction
- Aero-Thermo-Elastic Coupling Description
- Analysis Results
- Sensitivity Analysis and Optimization
- Sensitivity Analysis and Optimization Results
- Conclusions and Future Works

CONCLUSIONS

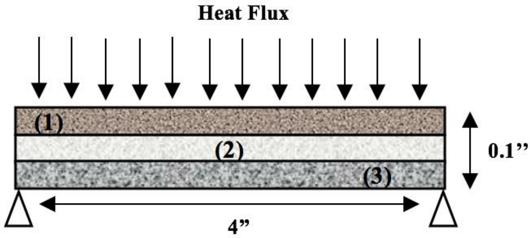
- ✓ **Validated** the structural solver's **thermal** and **thermo-elastic** analysis capability.
- ✓ **Developed/Validated** a **3D** transient **aero-thermo-elastic** analysis platform with a **weak coupling** approach using:
  - ✓ flow solver **NSU3D** with Mesh Deformation Capability
  - ✓ thermo-elastic capability from **AStrO**
  - ✓ FSI module
- ✓ **Verified** the **thermo-elastic adjoint and tangent** sensitivities.
- ✓ Demonstrated **standalone thermo-elastic optimization.**
- ✓ **Developed/Verified** the **aero-thermo-elastic adjoint and tangent** sensitivities.
- ✓ Demonstrated preliminary **aero-thermo-elastic optimization.**

CONCLUSIONS
Main Contributions

- ✓ Researched, developed, and validated the **aero-thermo-elastic analysis** coupling.
- ✓ Formulated, implemented, and tested the **coupled aero-thermo-elastic sensitivities.**
- ✓ Applied the verified sensitivities to preliminary **aero-thermo-elastic optimization** problems.
- ✓ Results from this work has been published in the following:
  - ✓ **AIAA 2020-1449 , SciTech 2020, January 2020**
  - ✓ **AIAA 2020-3138, Aviation 2020, June 2020.**
  - ✓ Manuscript accepted and under publication by **AIAA Journal,** as of **June 2021.**
  - ✓ Abstract submitted to **SciTech 2022, January 2022.**

FUTURE WORKS

- Further investigate aero-thermo-elastic panel optimization problems.
  - Combined material/flow design variables
  - Multi-layered panel constructions



- Further development of the coupled aero-thermo-elastic sensitivities.
- Further development of the fluid solver.
- Further development of the structural solver.
- Further development of the FSI module.
- Adaptive coupling time step size.
- Uncertainty quantification and reduced order modeling.

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THANK YOU!!!

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