Multidisciplinary Adjoint-based Design Optimization Techniques for Helicopter Rotors

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Outline

- Introduction
- Motivation & Objective
- Hybrid CFD/CAA Analysis, and Adjoint Sensitivity for Flexible Rotors
- Results
- Conclusions

Introduction

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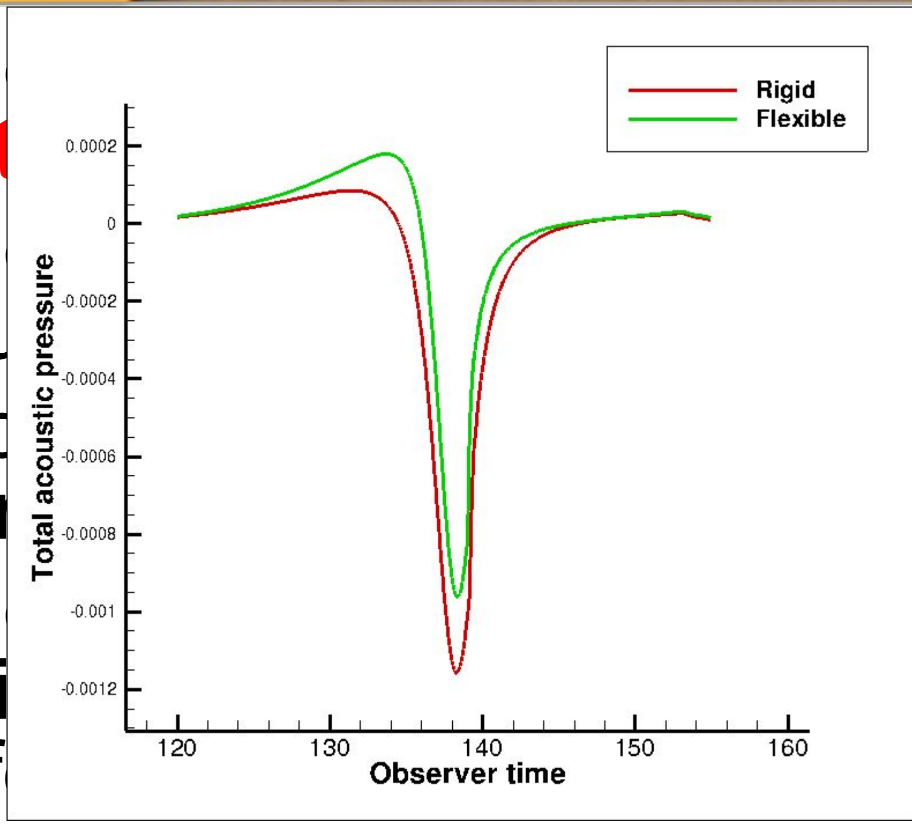
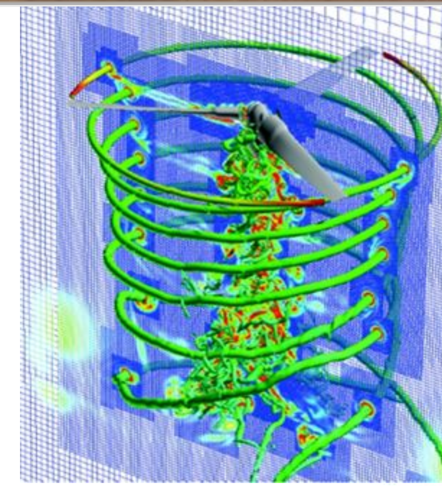
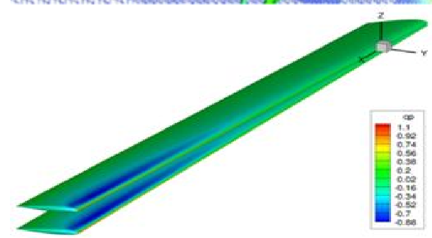
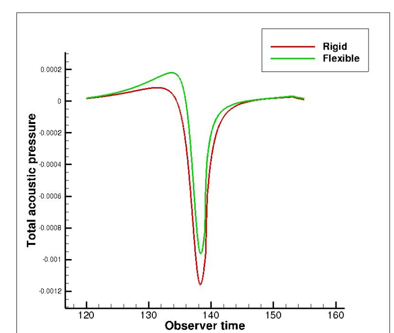


- Need for flexible *aero-acoustic* coupled adjoint optimization
  - Blade flexibility affects noise signature







Motivation & Objective

- Limited high-fidelity multidisciplinary optimization
  - Aerodynamics, structural mechanics, aeroacoustics
- Objective
  - Enable blade shape (and other) optimization to minimize far-field acoustic signature
  - Develop coupled near-field/far-field acoustic analysis and sensitivity capability for flexible rotors
  - Use to perform time-dependent optimization for far-field acoustic objectives
  - Demonstrate multidisciplinary capability
    - Combine aeroelastic and aeroacoustic adjoint

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Aerodynamic Solver

NSU3D

- 3D unstructured finite-volume RANS solver.
- 2^{nd} –order accurate in space and time.
  - Fully implicit discretization solved using Newton's method at each time-step
- Central differencing with Matrix dissipation.
- One equation Spalart-Allmaras turbulence model.
- Deforming mesh capability.
  - Linear elasticity model to propagate surface deformations to interior (cyclic pitching/design changes/structural deformations).
- MPI parallelization with proven scalability up to 30,000 processors.

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Structural Analysis: Beam Model

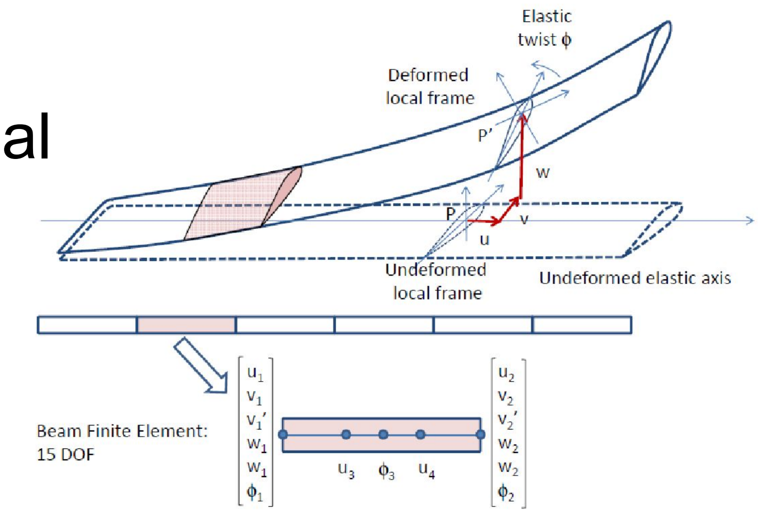
- Hodges-Dowell type finite element based solver
- 15 degrees of freedom (flap, lag, axial and torsion)
- First order system:

where, $\mathbf{J} = [I]\,\dot{\mathbf{Q}} + [A]\,\mathbf{Q} - \mathbf{F} = 0$

$\mathbf{Q} = [\mathbf{q}, \dot{\mathbf{q}}]^T$

- **J** = Residual of structural equation
- **q** = blade dof (flap, slope etc)
- **F** = beam forcing

Beam FEM model



Comparison of Hart-II Natural Frequencies

Modes	Present Model	UMARC	DLR
Flap 1	1.104	1.112	1.125
Flap 2	2.802	2.843	2.835
Flap 3	5.010	5.189	5.168
Torsion 1	3.878	3.844	3.845

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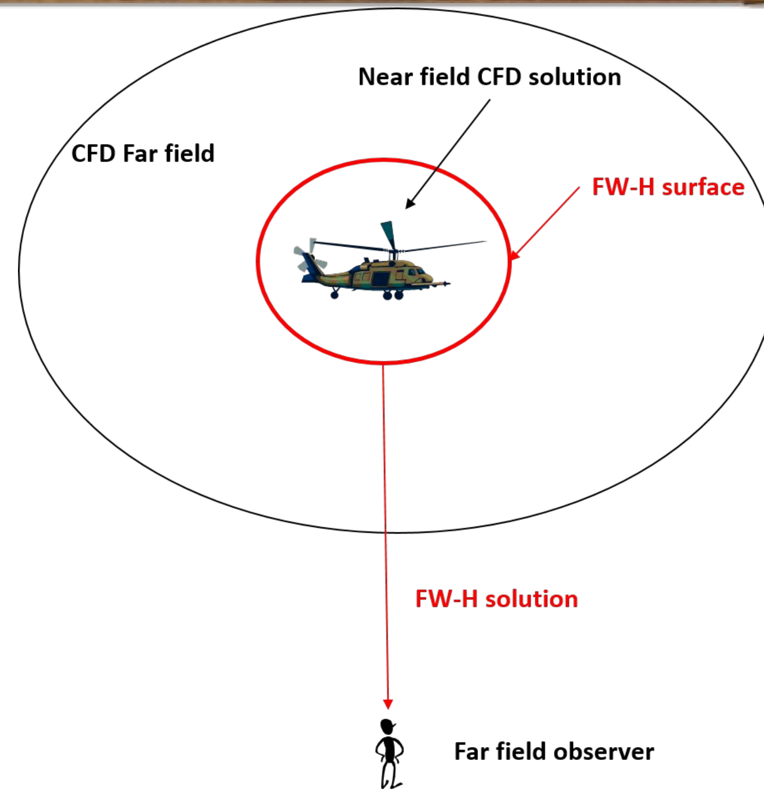
Acoustic propagation: FW-H

FW-H:

- Farassat's Formulation 1A
- Source-time dominant algorithm.
- Linear pressure interpolation at the observer.
- Quadrupole term neglected.
- Use aeroelastically converged flow and mesh data



$$U_i = \left(1 - \frac{\rho}{\rho_o}\right) v_i + \frac{\rho u_i}{\rho_o}$$

$$L_i = p' n_j + \rho u_i (u_n - v_n)$$

$$U_n = U_i n_i$$

$$M_r = M_i r_i$$

$$L_r = L_i r_i$$

$$L_M = L_i M_i$$

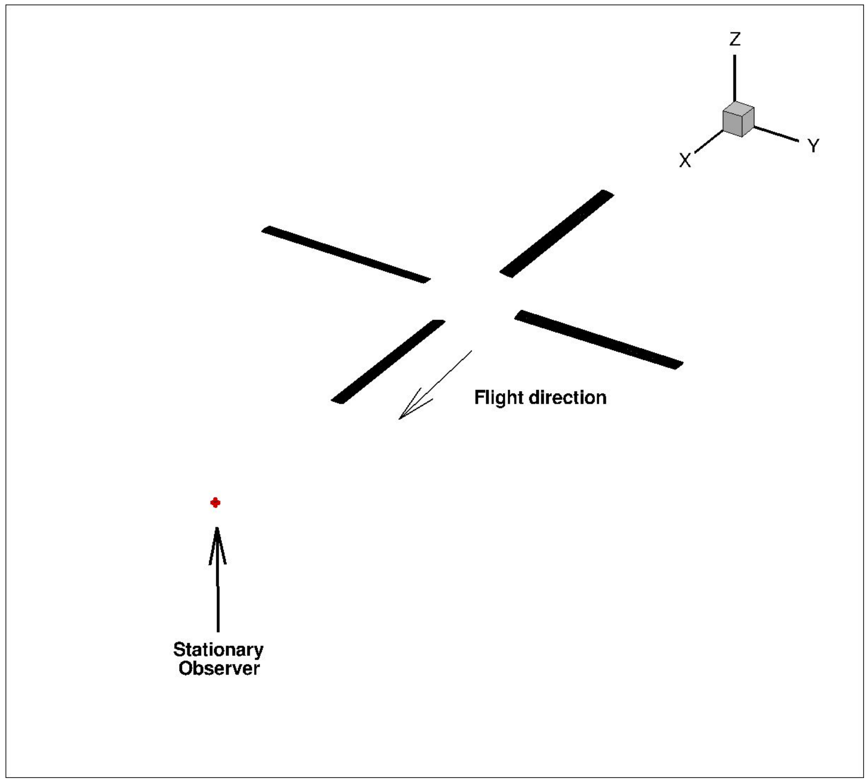
$$K = r\dot{M}_r + c_o (M_r - M^2)$$

$$4\pi p'(\mathbf{y},t) = 4\pi p'_T(\mathbf{y},t) + 4\pi p'_L(\mathbf{y},t)$$

$$4\pi p'_T(\mathbf{y},t) = \int_{f=0} \left[\frac{\rho_o (\dot{U}_n + U_{\dot{n}})}{r(1-M_r)^2}\right]_{ret} dS + \int_{f=0} \left[\frac{\rho_o U_n K}{r^2(1-M_r)^3}\right]_{ret} dS$$

$$4\pi p'_L = \frac{1}{c_o}\int_{f=0} \left[\frac{\dot{L}_r}{r(1-M_r)^2}\right]_{ret} dS + \int_{f=0} \left[\frac{L_r - L_M}{r^2(1-M_r)^2}\right]_{ret} dS + \frac{1}{c_o}\int_{f=0} \left[\frac{L_r K}{r^2(1-M_r)^3}\right]_{ret} dS$$

Acoustic Problem setup



Flexible Hart-II rotor in trimmed forward flight.

$M_\infty = 0.095\ - M_{tip} = 0.638$

Stationary observer in the plane of the rotor 2R from rotor hub $\psi = 180$deg.

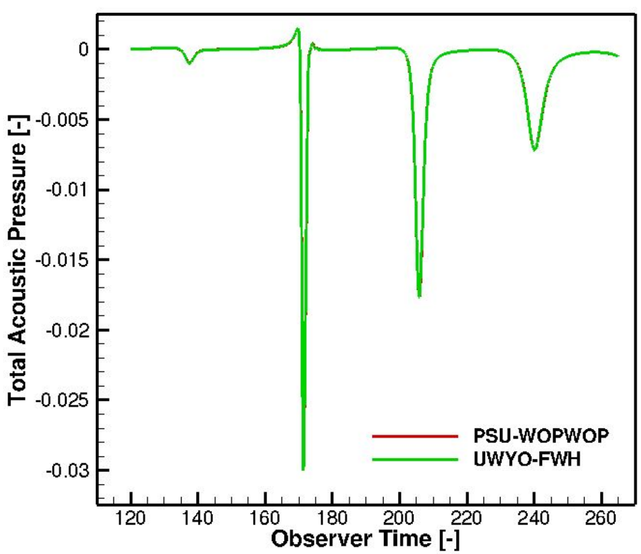
Solid wall acoustic integration – Blade surface is acoustic surface

FW-H validation

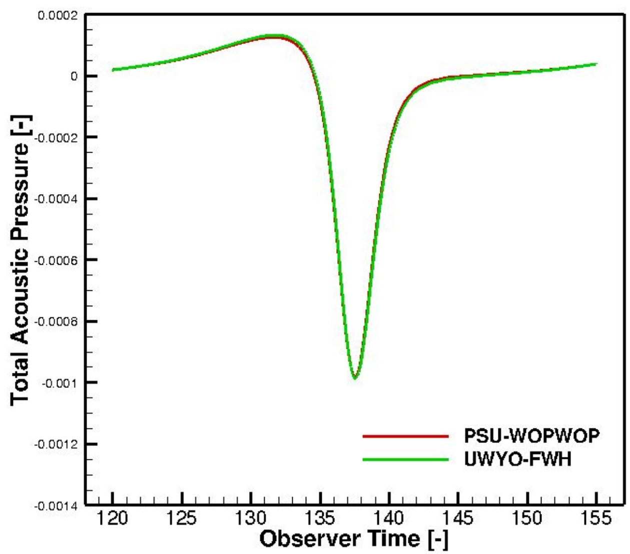
RANS based NSU3D CFD code provides input to FW-H acoustic propagation module.

Validation against legacy PSU-WOPWOP FW-H code

Observer time history



Optimization window



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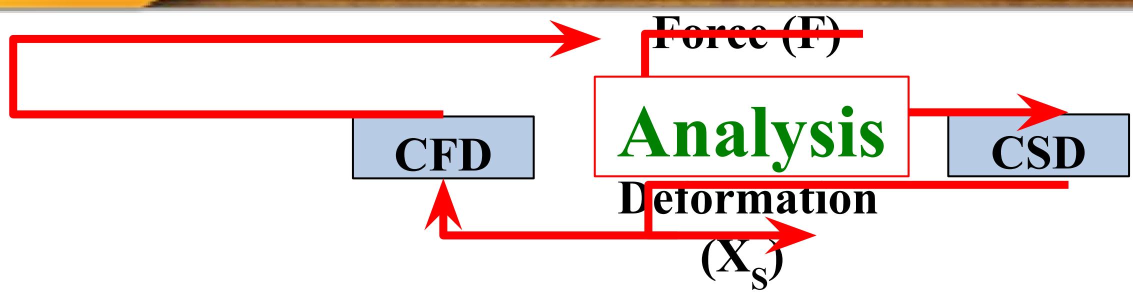
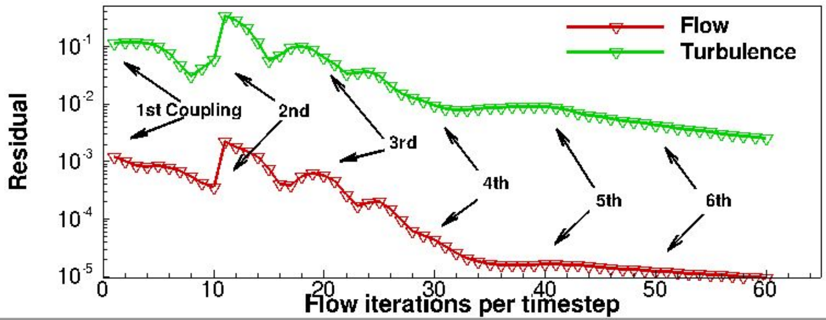
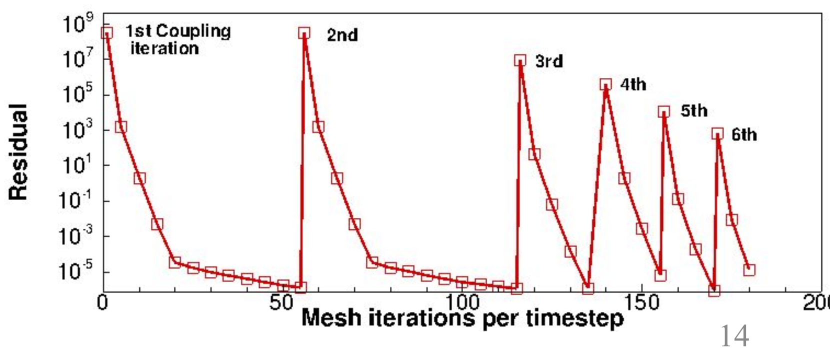
Fully coupled aerostructural analysis



Mesh deformation	$\mathbf{G}(\mathbf{x}, \mathbf{D}) = \mathbf{0}$
Pitching	$\mathbf{S}^{\theta}(\mathbf{x}_{s\theta}, \mathbf{x}_s, \mathbf{D}) = \mathbf{0}$
Mesh deformation	$\mathbf{G}(\mathbf{x}_{\theta}, \mathbf{x}_{s\theta}(\mathbf{Q}, \mathbf{D})) = \mathbf{0}$
Azimuth rotation	$\mathbf{S}^{\Psi}(\mathbf{x}_p, \mathbf{x}_{\theta}) = \mathbf{0}$
CFD	$\mathbf{R}(\mathbf{u}, \mathbf{x}_p) = \mathbf{0}$
Interpolation	$\mathbf{S}(\mathbf{F}_B, \mathbf{Q}, \mathbf{F}(\mathbf{x}_p, \mathbf{u})) = \mathbf{0}$
CSD	$\mathbf{J}(\mathbf{Q}, \mathbf{F}_B) = \mathbf{0}$
Interpolation	$\mathbf{S}'(\mathbf{x}_s, \mathbf{Q}) = \mathbf{0}$





Fully coupled aerostructural sensitivity: Tangent

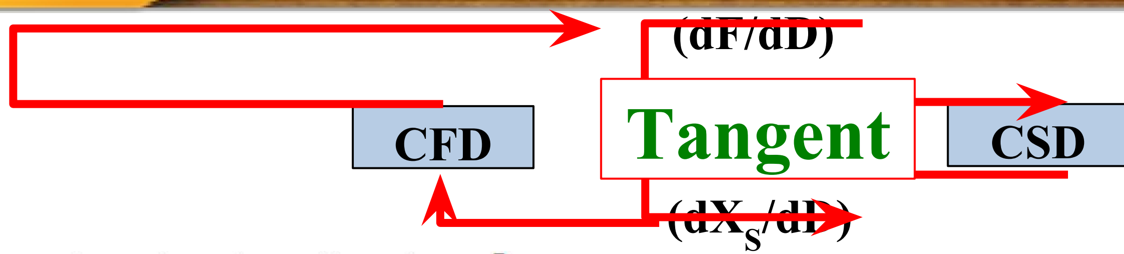
- Time-dependent objective function:

$$L = L(\mathbf{U}(\mathbf{D}), \mathbf{x}(\mathbf{D}))$$

- General expression for forward linearization of objective function w.r.t design variables:

$$\frac{dL}{d\mathbf{D}} = \begin{bmatrix} \frac{\partial L}{\partial \mathbf{x}} & \frac{\partial L}{\partial \mathbf{U}} \end{bmatrix} \begin{bmatrix} \frac{d\mathbf{x}}{d\mathbf{D}} \\ \frac{d\mathbf{U}}{d\mathbf{D}} \end{bmatrix}$$

Fully coupled aerostructural sensitivity: Tangent



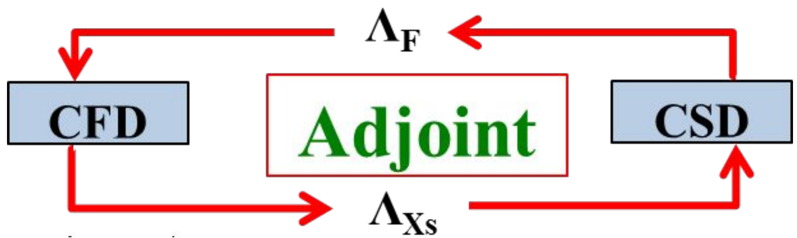
$$\left[\begin{array}{c} \partial G \\ \cdots \\ \partial Q \quad \partial x_s \end{array}\right]
\begin{bmatrix} \frac{dx}{dD} \\ \frac{dx_{s\theta}}{dD} \\ \frac{dx_\theta}{dD} \\ \frac{dx_p}{dD} \\ \frac{du}{dD} \\ \frac{dF_b}{dD} \\ \frac{dQ}{dD} \\ \frac{dx_s}{dD} \end{bmatrix}
=
\begin{bmatrix} -\frac{\partial G}{\partial D} \\ -\frac{\partial S^\theta}{\partial D} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

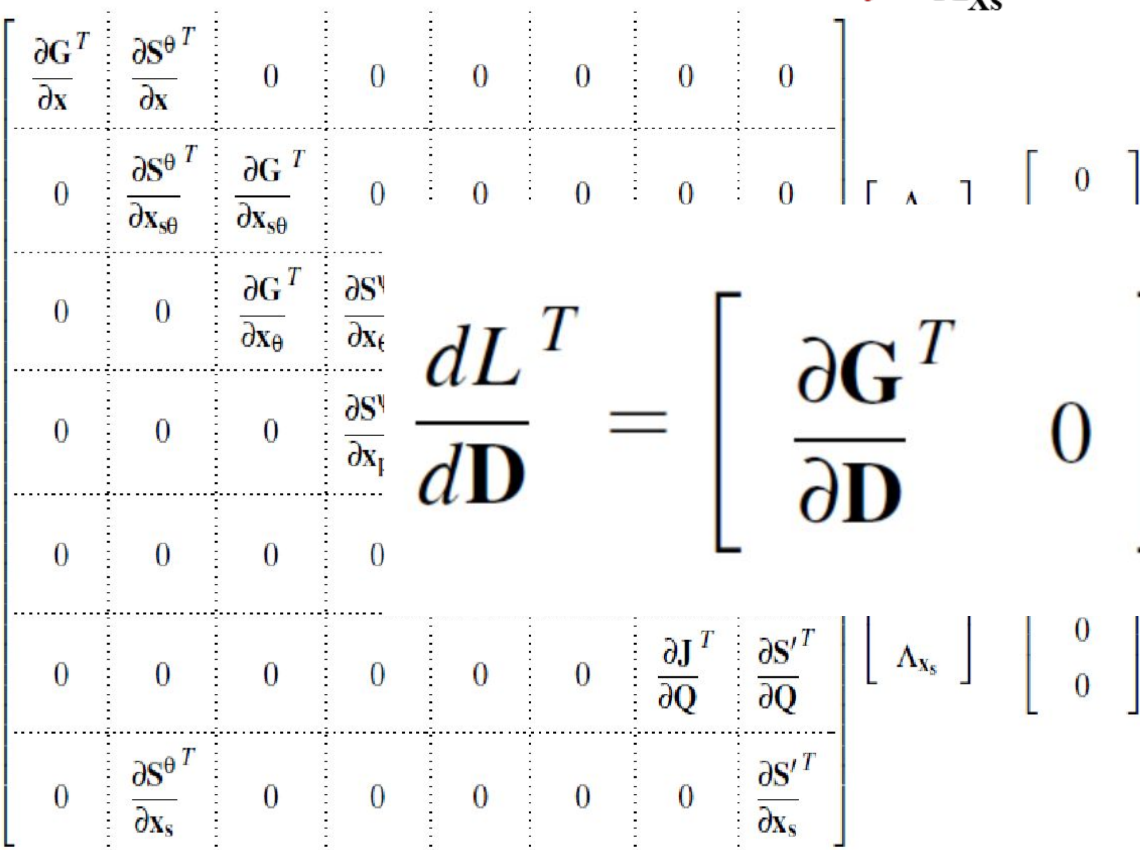
$$\mathbf{G}(\mathbf{x},\mathbf{D}) = 0$$
$$\mathbf{S}^\theta(\mathbf{x}_{s\theta},\mathbf{x}_s,\mathbf{D}) = 0$$
$$\mathbf{G}(\mathbf{x}_\theta,\mathbf{x}_{s\theta}(\mathbf{Q},\mathbf{D})) = 0$$
$$\mathbf{S}^\psi(\mathbf{x}_p,\mathbf{x}_\theta) = 0$$
$$\mathbf{R}(\mathbf{u},\mathbf{x}_p) = 0$$
$$\mathbf{S}(\mathbf{F}_B,\mathbf{Q},\mathbf{F}(\mathbf{x}_p,\mathbf{u})) = 0$$
$$\mathbf{J}(\mathbf{Q},\mathbf{F}_B) = 0$$
$$\mathbf{S}'(\mathbf{x}_s,\mathbf{Q}) = 0$$

Forward integration in time

Constraints to be satisfied at every timestep
Equations
Aeroelastic system dependent on D

Fully coupled aerostructural sensitivity: Adjoint





$$\frac{dL}{d\mathbf{D}}^T = \begin{bmatrix} \frac{\partial G}{\partial \mathbf{D}}^T & 0 \end{bmatrix} \begin{bmatrix} \Lambda_\mathbf{x} \\ \Lambda_\mathbf{u} \end{bmatrix}$$

Backward integration

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dependent

L

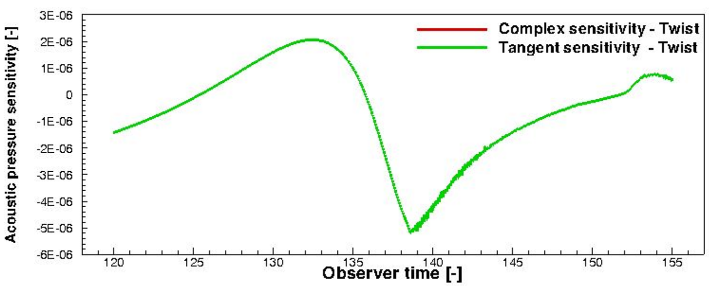
Independent of **D**

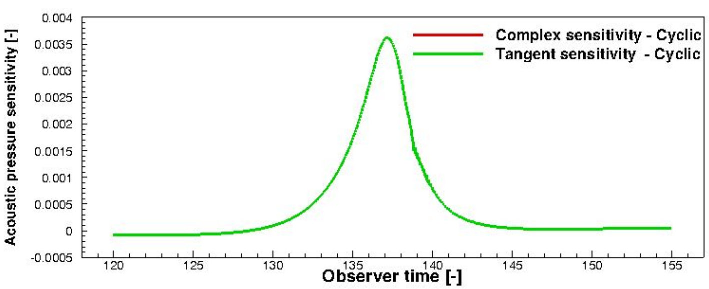
Acoustic sensitivity: verification

Tangent sensitivity: Tangent NSU3D solver provides forward aeroelastic flow and mesh sensitivity to tangent FW-H integration

Adjoint sensitivity: Adjoint FW-H code provides reverse sensitivity to NSU3D adjoint solver to perform aeroelastic backward time integration

Acoustic sensitivity time history





Agreement to 9 significant figures with complex step method

	Sensitivity (twist)
Complex	1.46204808**801**E-06
Tangent	1.46204806**875**E-06
Adjoint	1.46204808**794**E-06

	Sensitivity (cyclic)
Complex	-4.26987811**0954**E-04
Tangent	-4.26987811**0962**E-04
Adjoint	-4.26987811**0854**E-04

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  - Flexible HART-II
    - Trim optimization
    - Aeroacoustic optimization
    - Acoustically constrained torque minimization
  - Rigid UH60
    - Trim optimization
    - Aeroacoustic optimization
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Optimization Problems – HART-II

Objective: Minimize acoustic signature of HART-II rotor in trimmed forward flight.

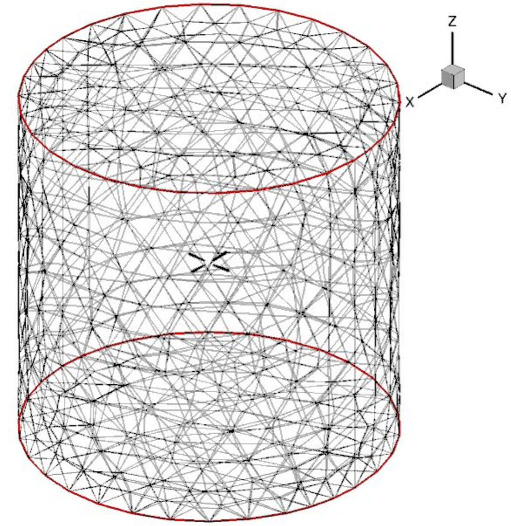
- Initial design: baseline HART-II rotor in trimmed forward flight
  - ➢ Initial trim formulated as separate optimization problem
- Aeroacoustic optimization
- Torque optimization with acoustic constraint
- SNOPT constrained optimizer
  - ➢ Enforces constraints directly
  - ➢ Requires adjoint calculation for each objective/constraint
- Optimization cost: 96 hours wall-clock time on 1024 cores
- 650Gb disk storage

HART-II rotor in Forward Flight

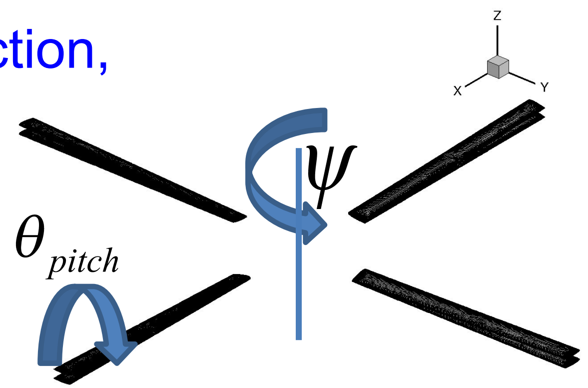
- 4 bladed Hart-II rotor in forward flight:
  - M_{tip} = 0.64; 1040 RPM; μ=0.15 (M_∞~0.1); α=5.4^O
- CFD/CSD specifications:
  - 2.32 million grid nodes (prisms, pyramids, tets)
  - 20 beam elements per blade
  - 2 rotor revs, $\Delta t=2^O$
    - 6 coupling iterations per timestep
  - Objective/constraints accumulated over second rotor revolution
- Design variables
  - 10 Hicks-Henne bump functions per blade section, 9 blade sections (90). Root and tip twist.
  - Control Inputs:
    - Collective (θ_O) and Cyclics (θ_{1c}, θ_{1s})

$$\theta_{pitch} = \theta_O + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi$$

  - 95 design parameters total



CFD flow domain



Optimization Problems – HART-II

Trim optimization

$$\min L_{THRUST}$$
subject to
$$L_{LATERAL} = 0$$
$$w.r.t.\ \mathbf{D}_{\mathbf{pitch}}$$

Aeroacoustic optimization

$$\min p'_{RMS}$$
subject to
$$L_{THRUST} = 0$$
$$L_{LATERAL} = 0$$
$$w.r.t.\ \mathbf{D}$$

Acoustically constrained Torque optimization

$$\min L_{TORQUE}$$
subject to
$$L_{THRUST} = 0$$
$$L_{LATERAL} = 0$$
$$p'_{RMS} = p'_{RMS_{TARGET}}$$
$$w.r.t.\ \mathbf{D}$$

Optimization Problems – HART-II

Acoustic
objective/constraint

$$p'_{RMS} = \sqrt{\frac{\sum_{i=1}^{N_{sample}} p'^2(\mathbf{D})}{N_{sample}}}$$

$p'_{RMS_{TARGET}}$ yields 2dB OSPL reduction

Aerodynamic
constraints

$$L_{THRUST} = \frac{1}{N}\left(\sum_{i=1}^{N}\left(C_T^i - C^i_{T_{AVERAGE}}\right)\right)^2$$

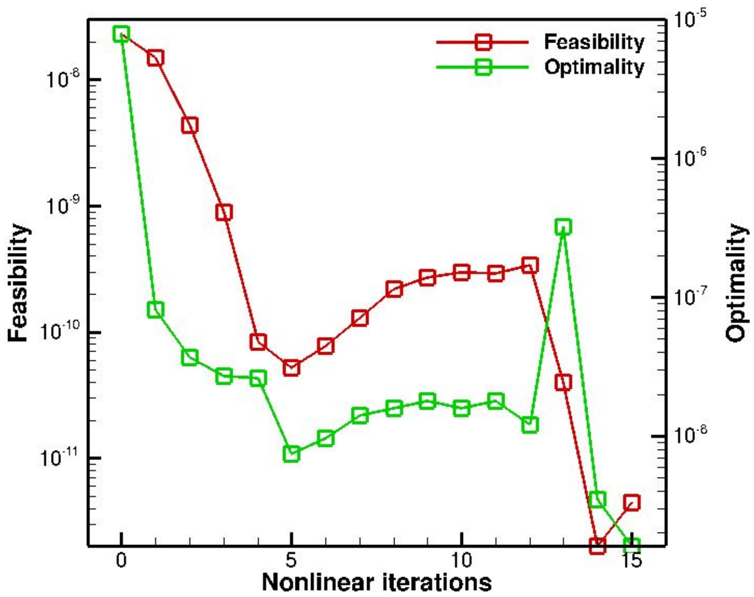
$$L_{LATERAL} = \frac{1}{N}\left[\left(\sum_{i=1}^{N} C^i_{M_x}\right)^2 + \left(\sum_{i=1}^{N} C^i_{M_y}\right)^2\right]$$

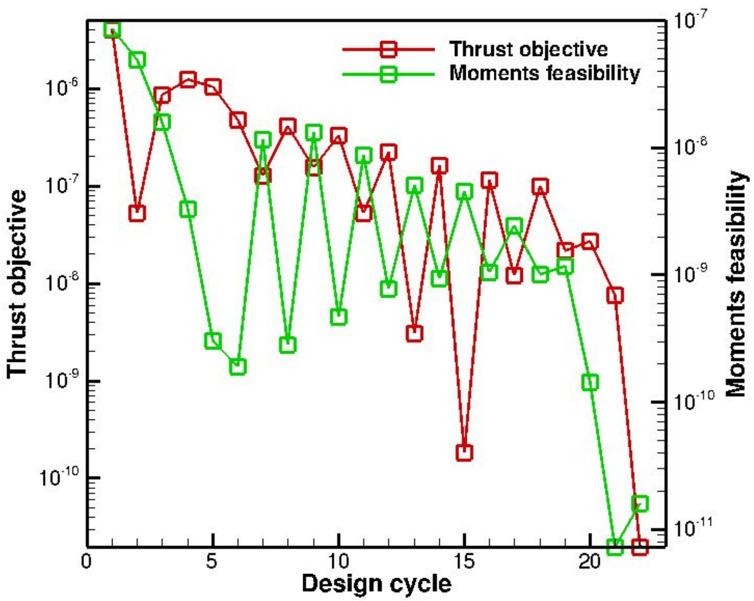
$$L_{TORQUE} = \frac{1}{N}\sum_{i=1}^{N}\left(C_Q^i\right)^2$$

$$C^i_{T_{AVERAGE}} = 0.0044$$

Optimization Problems – HART-II

Trim optimization



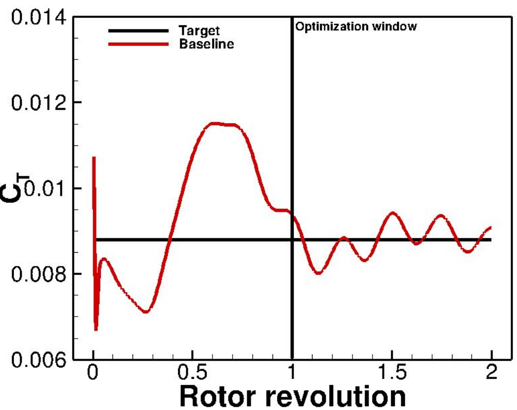


- ✓ Convergence in 15 non linear iterations
- ✓ Only 3 design variables: 1 collective, 2 cyclics

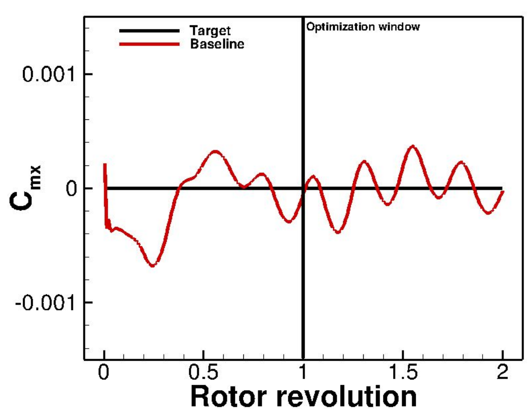
Optimization Problems – HART-II

Trim optimization

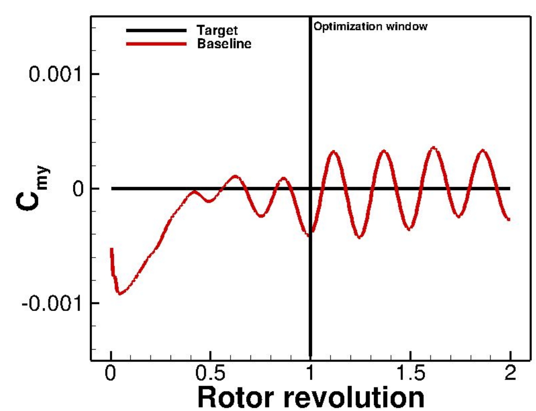
Thrust



Rolling moment



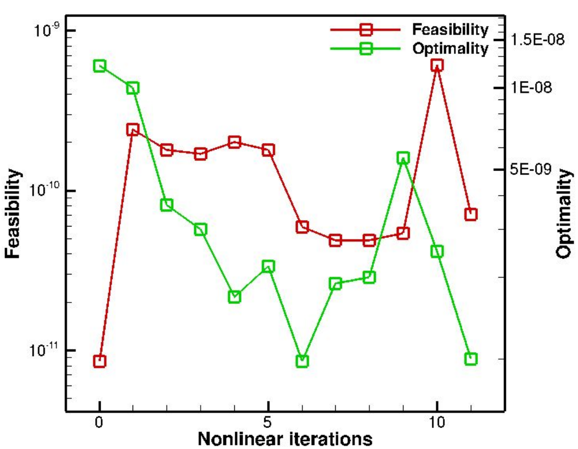
Pitching moment

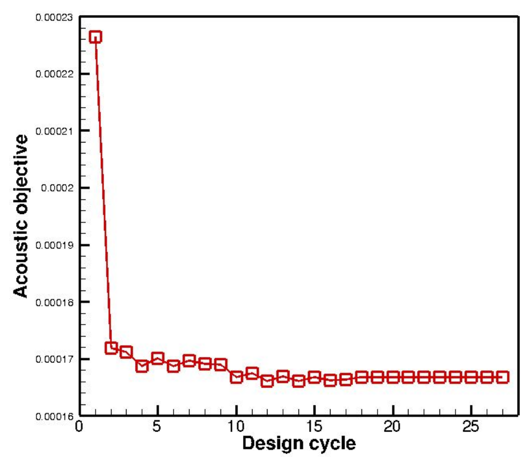


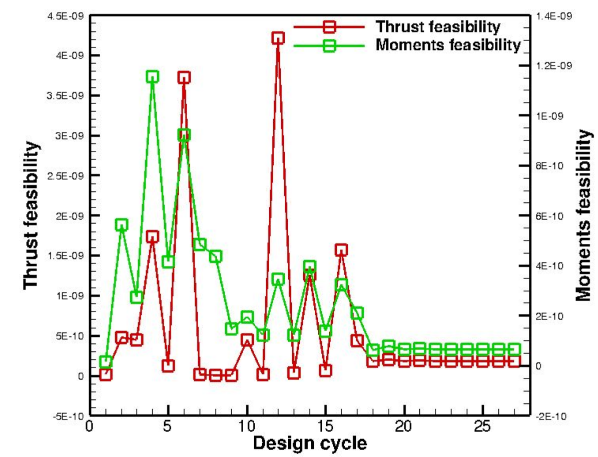
- ✓ Average thrust equals target thrust
- ✓ Zero average lateral moments

Optimization Problems – HART-II

Aeroacoustic optimization





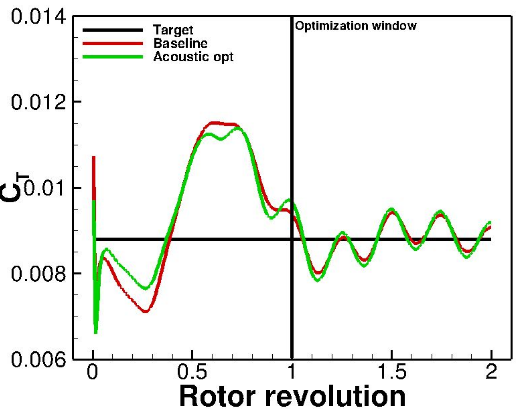


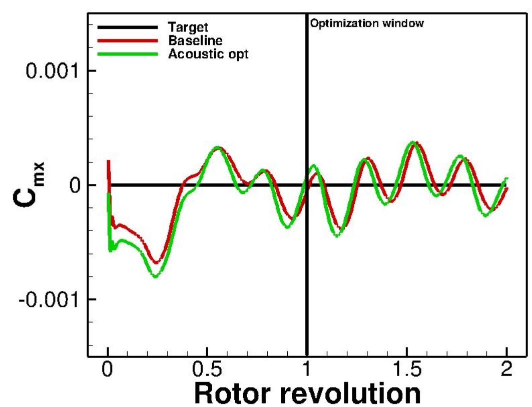
- ✓ Single objective, 2 constraints: 3adjoints
- ✓ 95 design variables
- ✓ Convergence in 11 non linear iterations
- ✓ Baseline OSPL reduced 2.6dB

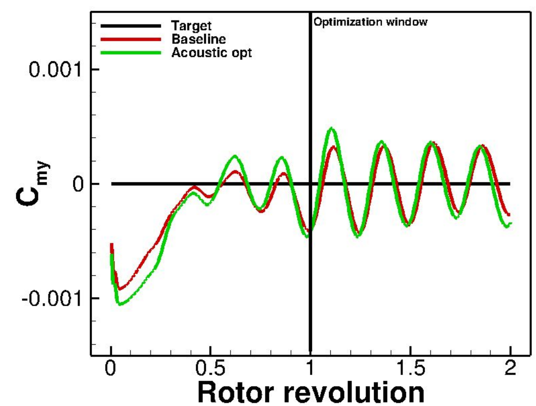
Optimization Problems – HART-II

Aeroacoustic optimization

Thrust | Rolling moment | Pitching moment





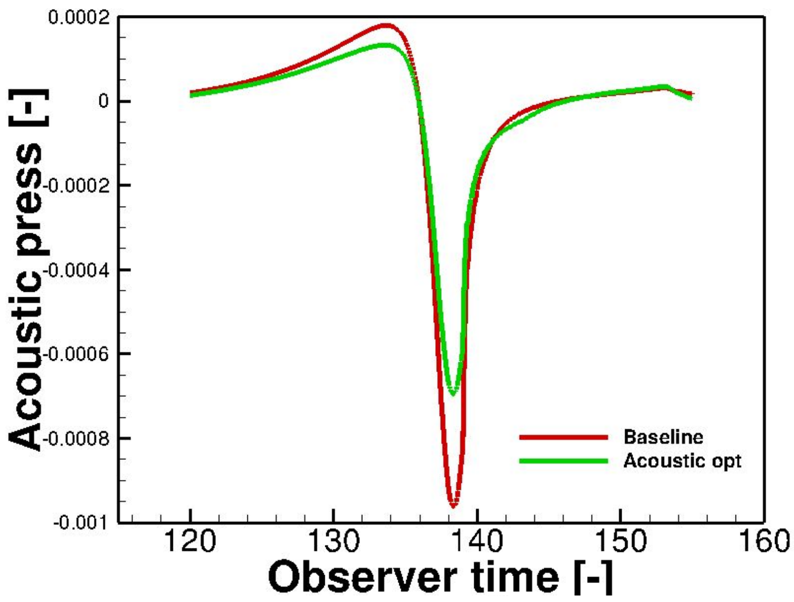


- ✓ Average thrust equals target thrust
- ✓ Zero average lateral moments
- ✓ Trimmed optimal design

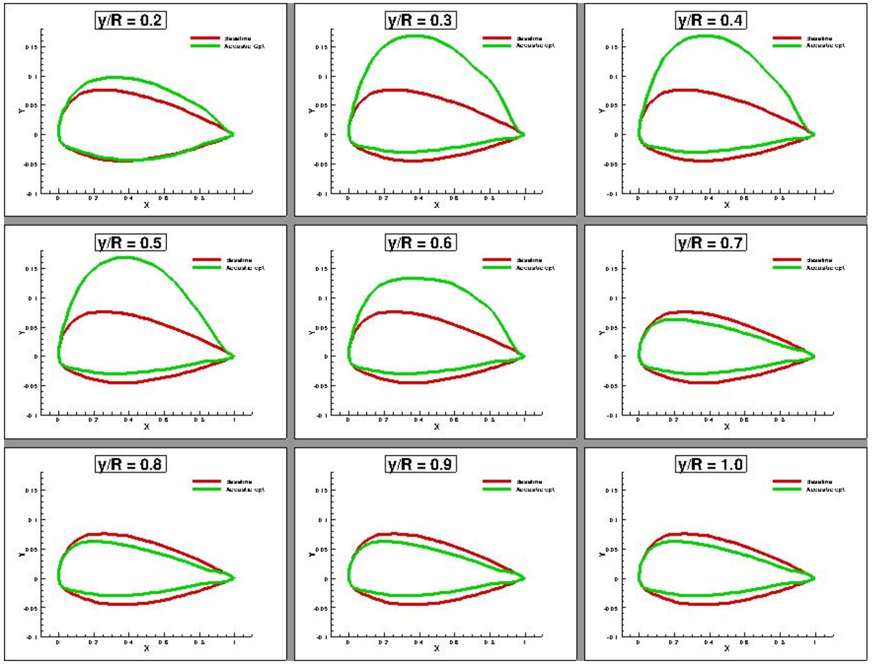
Optimization Problems – HART-II

Aeroacoustic optimization

Acoustic pressure time history



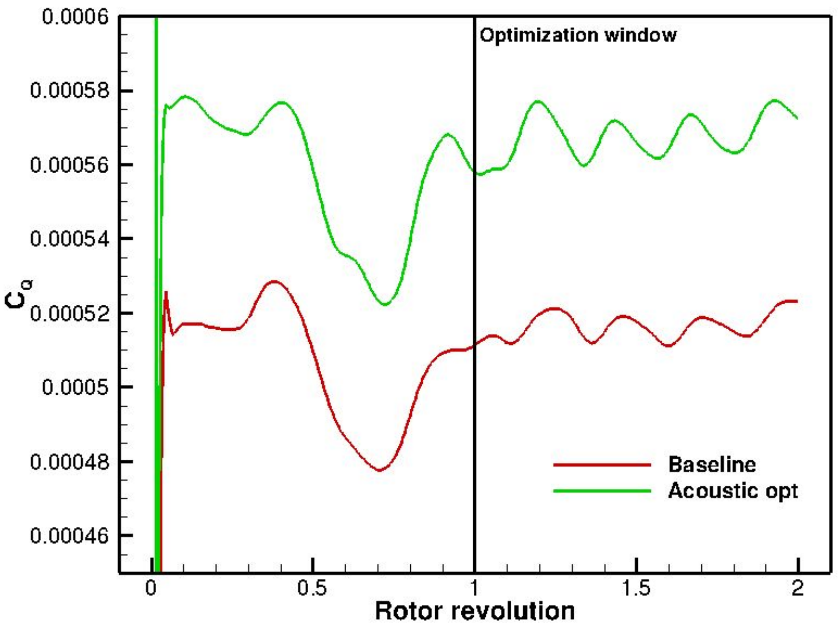
Optimized blade shapes



Optimization Problems – HART-II

Aeroacoustic optimization

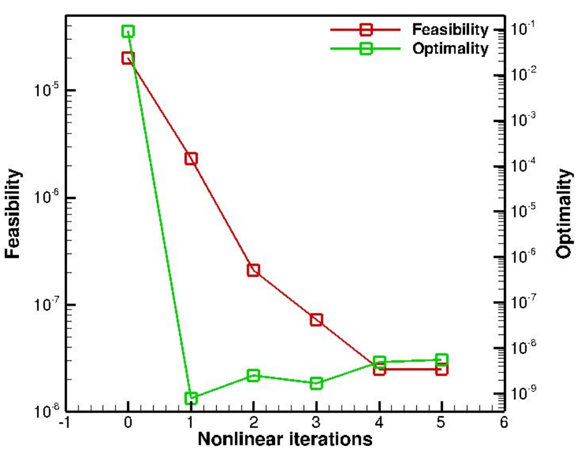
Torque time history

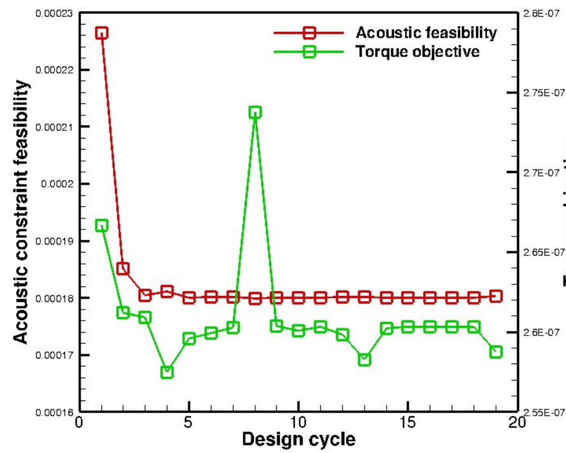


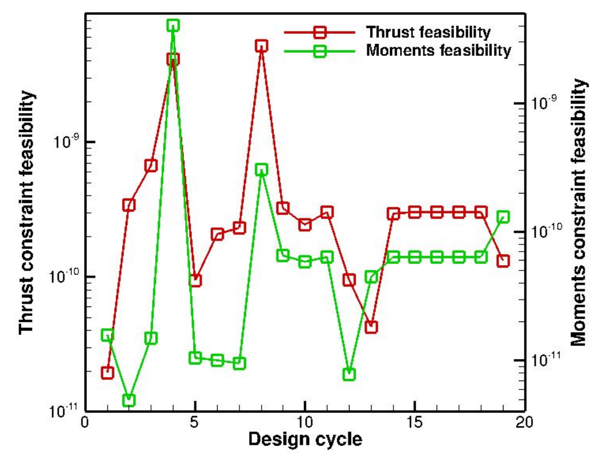
Noise reduction at observer achieved with strong performance penalty

Optimization Problems – HART-II

Acoustically constrained torque minimization





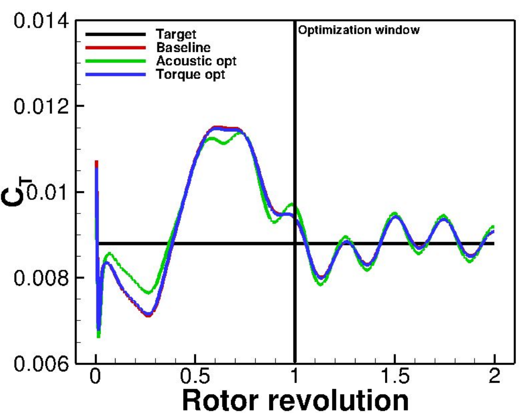


- ✓ 1 objective, 3 constraints: 4 adjoints
- ✓ 95 design variables
- ✓ Convergence in 5 non linear iterations
- ✓ 2.5% torque reduction and 2dB OSPL quieter rotor

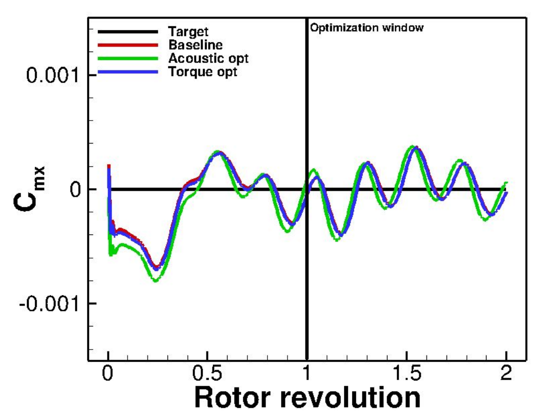
Optimization Problems – HART-II

Acoustically constrained torque minimization

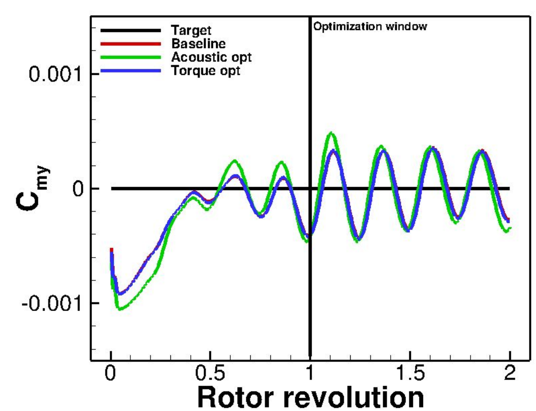
Thrust



Rolling moment



Pitching moment

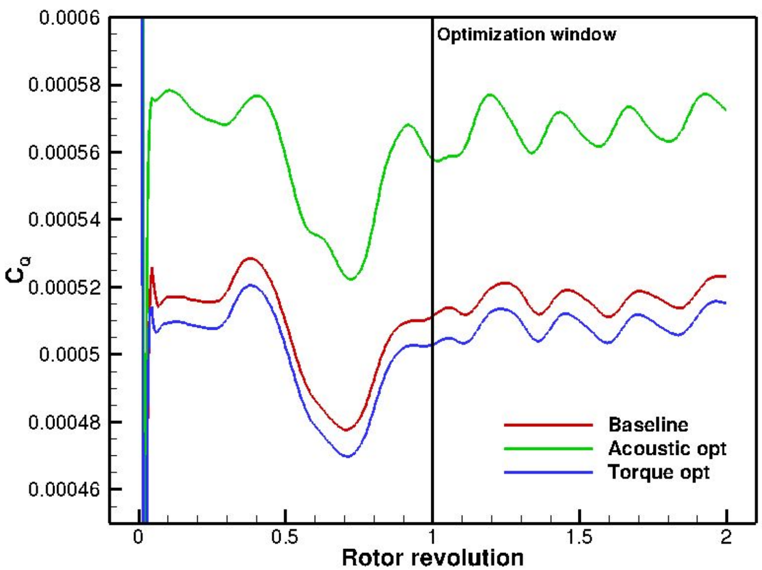


✓ Average thrust equals target thrust
✓ Zero average lateral moments
✓ Trimmed optimal design

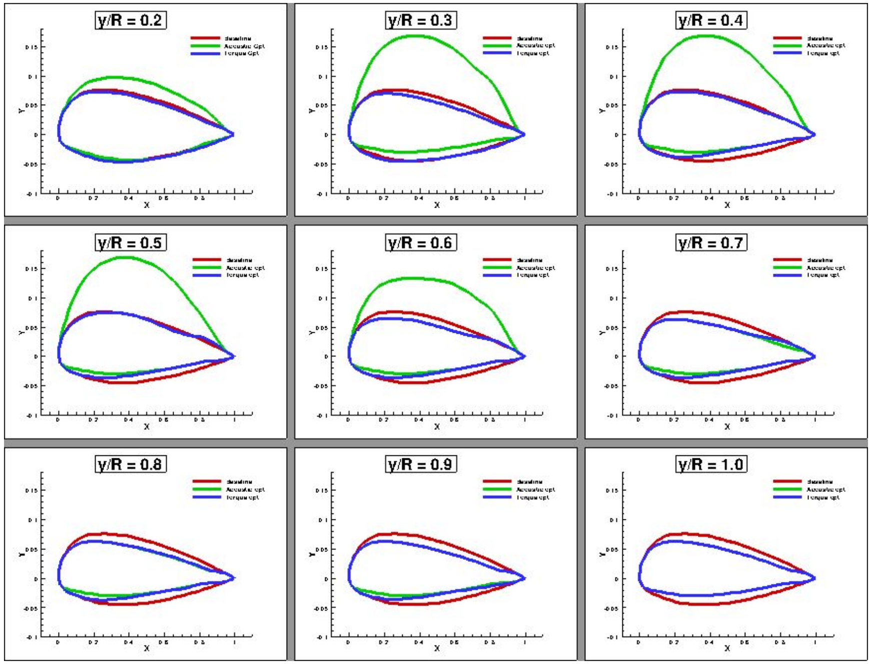
Optimization Problems – HART-II

Acoustically constrained torque minimization

Torque time history



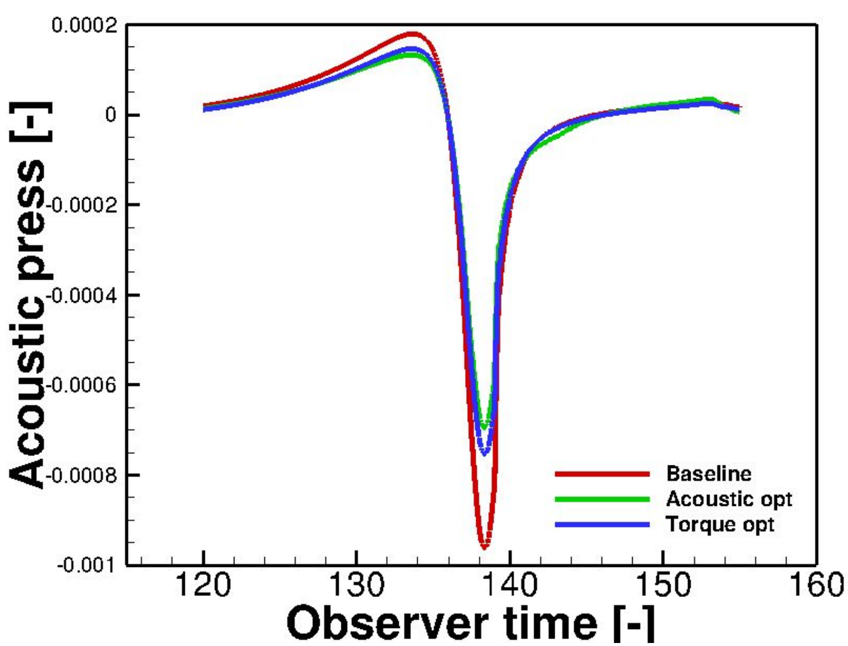
Optimized blade shapes



Optimization Problems – HART-II

Acoustically constrained torque minimization

Acoustic pressure time history

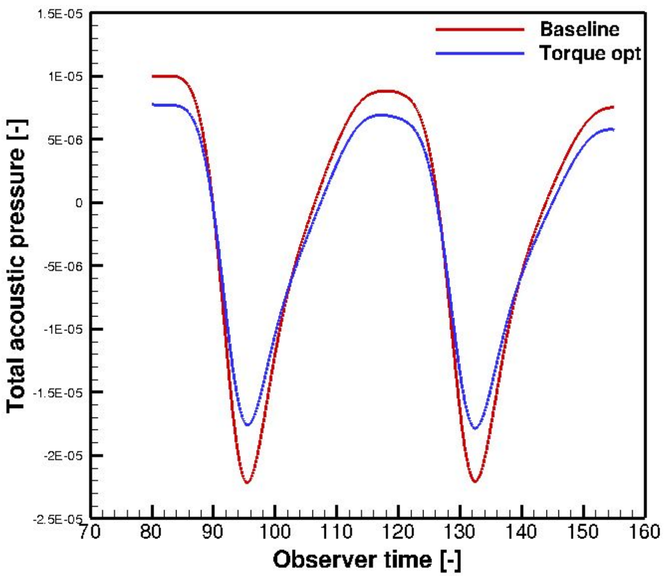


- **Acoustic constraint results in rotor 2dB OSPL quieter than baseline**
- **Noise minimized at different observer locations too**

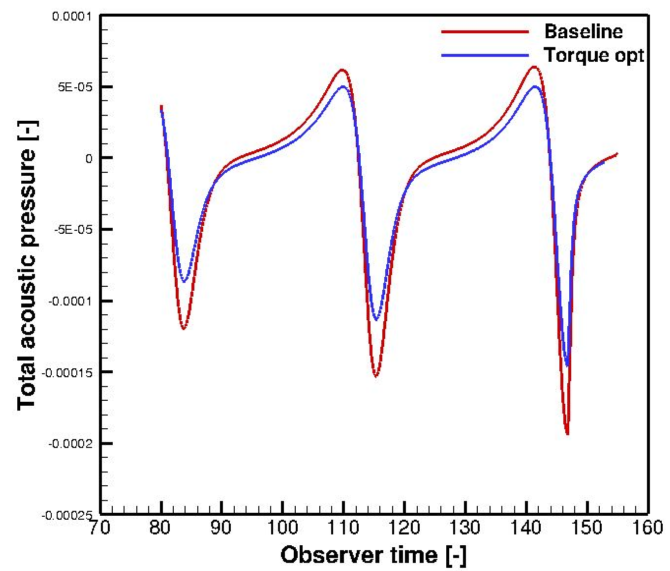
Optimization Problems – HART-II

Acoustically constrained torque minimization

Acoustic pressure time history
$\psi = 135$ deg



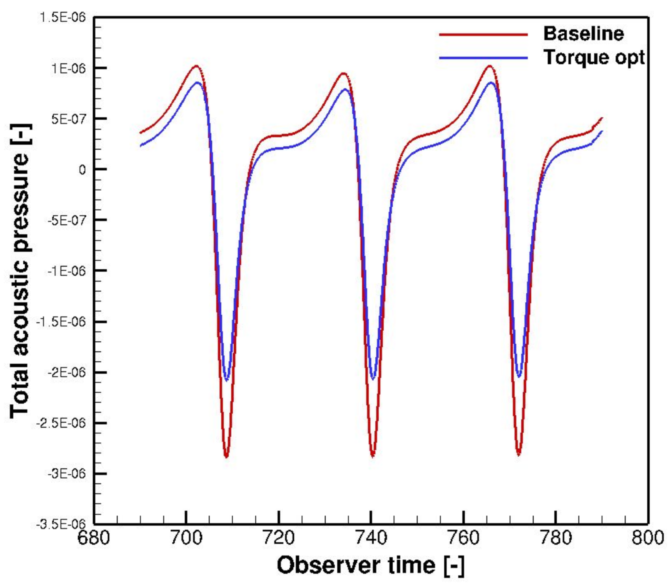
Acoustic pressure time history
$\psi = 315$ deg



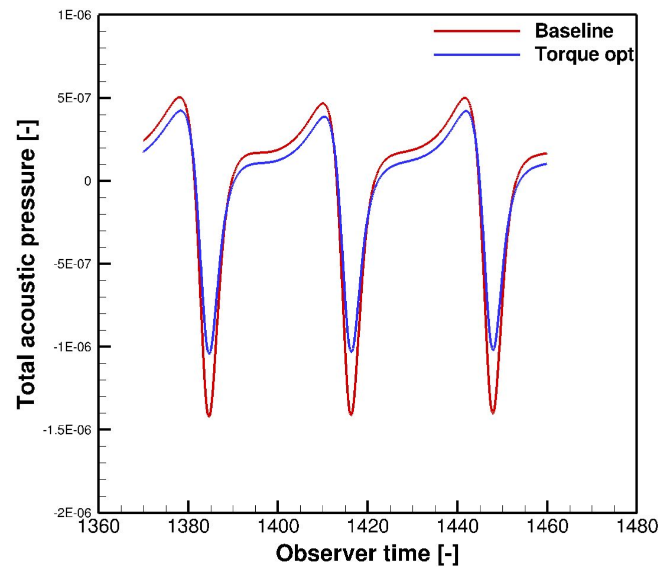
Optimization Problems – HART-II

Acoustically constrained torque minimization

Acoustic pressure time history
50R

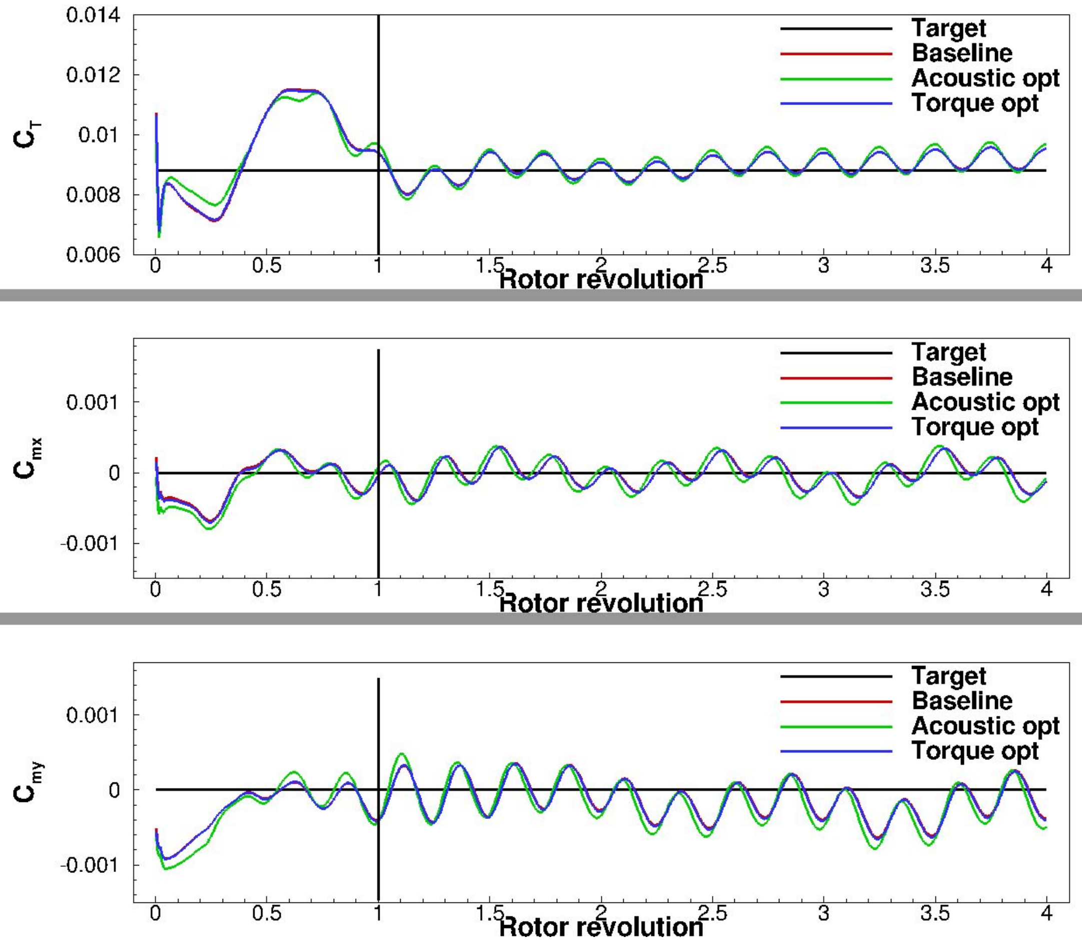


Acoustic pressure time history
100R



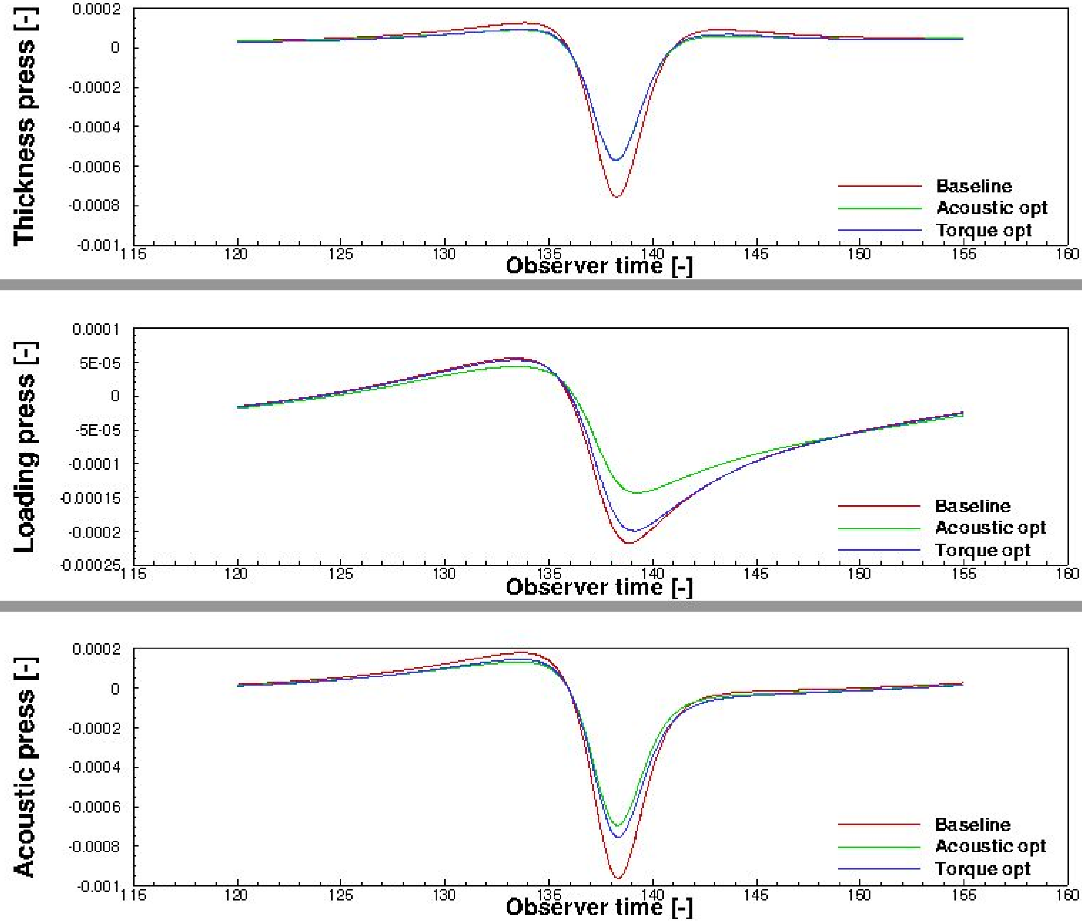
Optimization Problems – HART-II

Investigation over multiple rotor revolutions



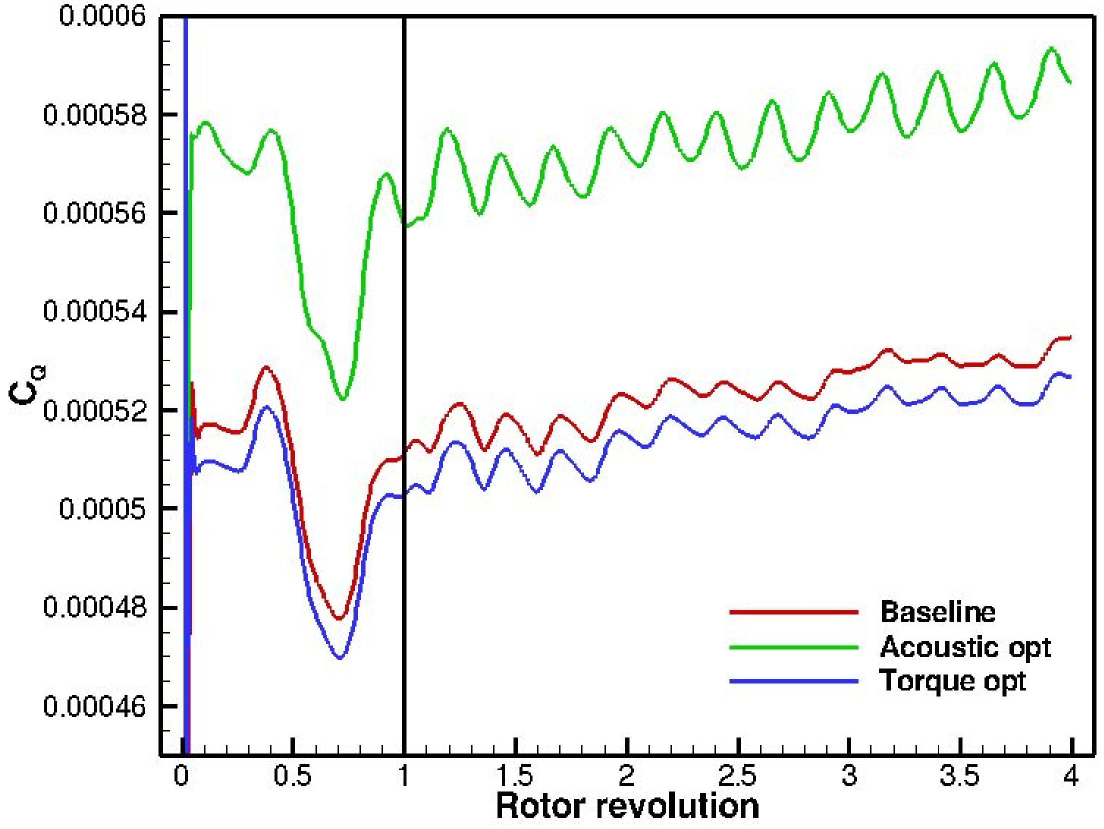
Optimization Problems – HART-II

Investigation over multiple rotor revolutions



Optimization problems

Investigation over multiple rotor revolutions



Optimization Problems – UH60

Objective: Minimize acoustic signature of rigid UH60 rotor in trimmed forward flight.

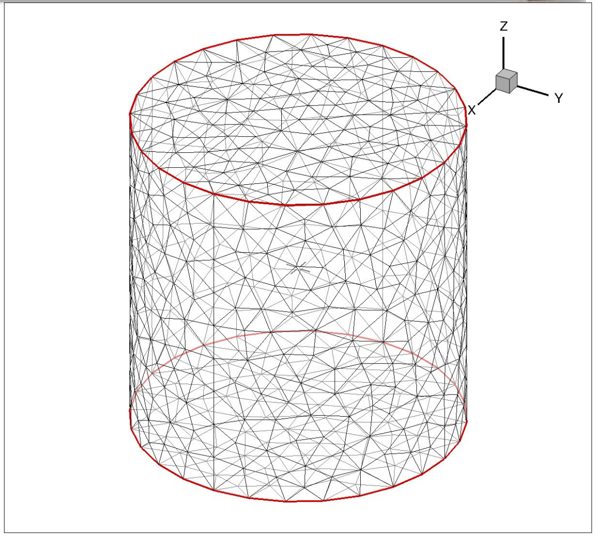
- Initial design: baseline UH60 rotor in trimmed forward flight
  - ➢ Initial trim formulated as separate optimization problem
- Aeroacoustic optimization (Wind tunnel formulation)
- SNOPT optimizer
  - ➢ Penalty function approach
  - ➢ Requires retrim after convergence
- Optimization cost: 66 hours wall-clock time on 3008 cores
- 2Tb disk storage

UH60 rotor in Forward Flight

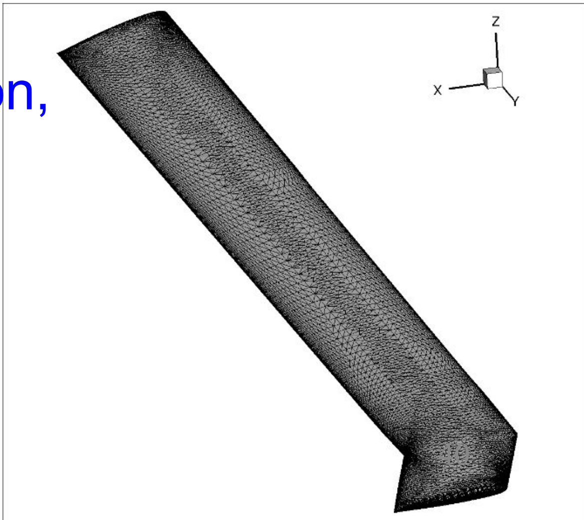
- 4 bladed UH60 rotor in forward flight (flight counter 8534)
  - M_{tip} = 0.64; 258 RPM; μ=0.368 (M_∞~0.236); α=-7.3^O
- CFD specifications:
  - 5.1 million grid nodes (prisms, pyramids, tets)
  - 2 rotor revs, $\Delta t = 2^O$
  - Objective/constraints accumulated over second rotor revolution
- Design variables
  - 10 Hicks-Henne bump functions per blade section, 11 blade sections. 11 twist spanwise sections. Taper, sweep and droop of tip section.
  - Control Inputs:
    - Collective (θ_O) and Cyclics (θ_{1c}, θ_{1s})

$$\theta_{pitch} = \theta_O + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi$$

  - 127 design parameters total



CFD flow domain



Optimization Problems – UH60

Trim optimization

$$min\ L_{THRUST}$$

$$\texttt{subject to}$$

$$L_{LATERAL} = 0$$

$$w.r.t.\ \mathbf{D_{pitch}}$$

Aeroacoustic optimization

$$min\ L_{UNC}$$

$$w.r.t.\ \mathbf{D}$$

$$L_{UNC} = L_{FWH} + 10L_{THRUST} + 100L_{LATERAL}$$

Optimization Problems – UH60

Acoustic objective

$$p'_{RMS} = \sqrt{\frac{\sum_{i=1}^{N_{sample}} p'^2(\mathbf{D})}{N_{sample}}}$$

Aerodynamic constraints

$$L_{THRUST} = \frac{1}{N}\left(\sum_{i=1}^{N}\left(C_T^i - C_{T_{AVERAGE}}^i\right)\right)^2$$

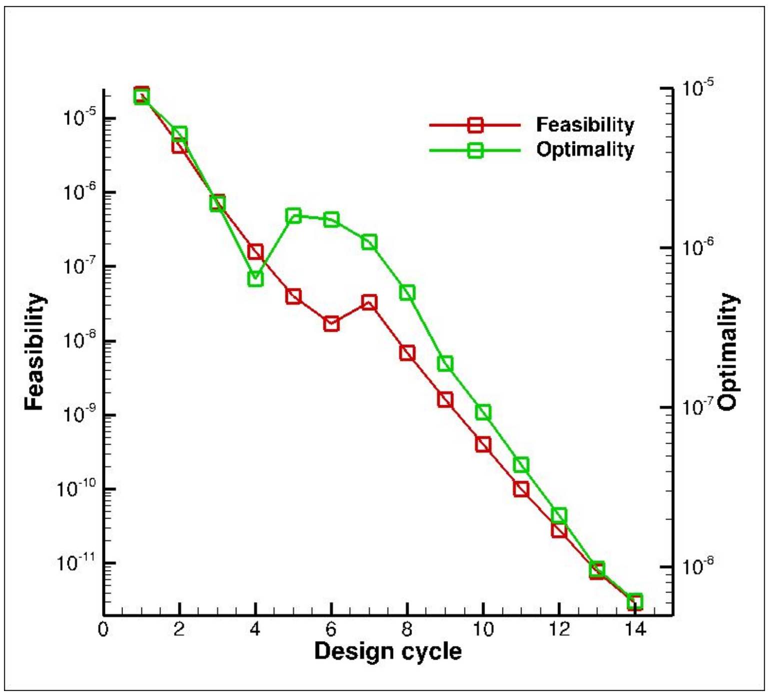
$$L_{LATERAL} = \frac{1}{N}\left[\left(\sum_{i=1}^{N}\left(C_{M_x}^i - C_{M_{x-average}}^i\right)\right)^2 + \left(\sum_{i=1}^{N}\left(C_{M_y}^i - C_{M_{y-average}}^i\right)\right)^2\right]$$

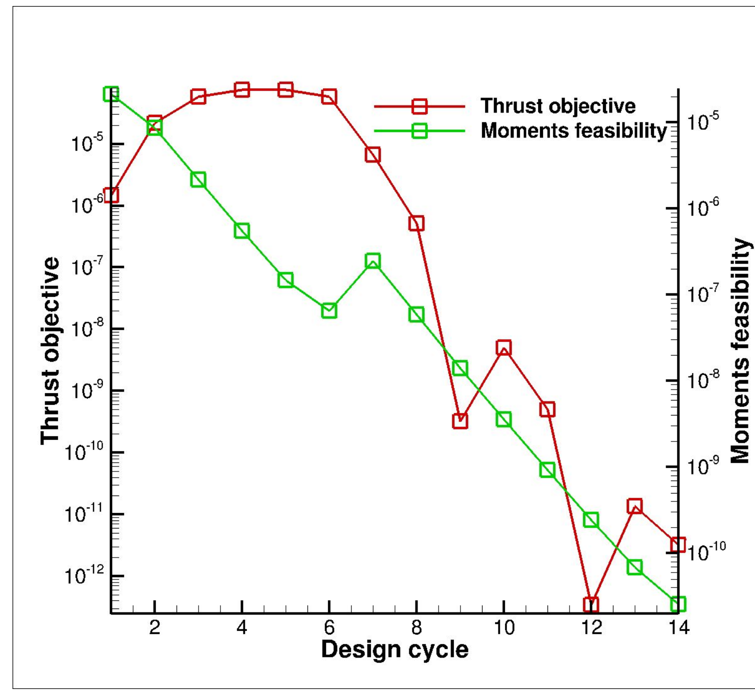
Flight counter 8534

$$C_{T_{AVERAGE}} = 0.0067$$
$$C_{M_{x-AVERAGE}} = 8.0535E-5$$
$$C_{M_{y-AVERAGE}} = -7.5640E-5$$

Optimization Problems – UH60

Trim optimization

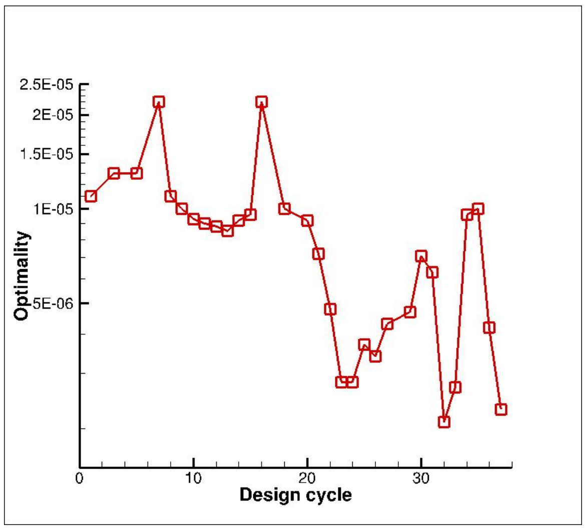


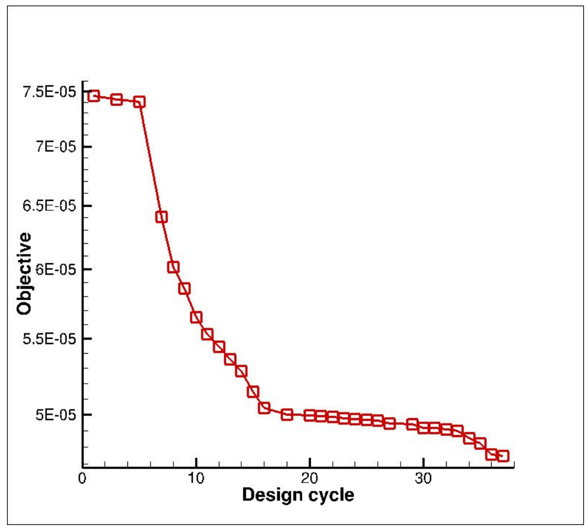


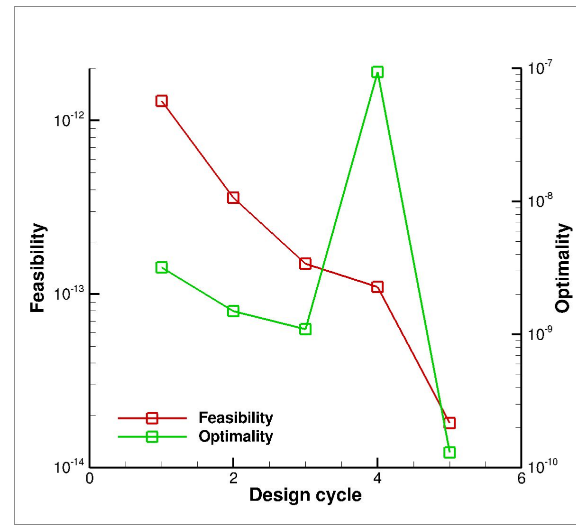
- ✓ Convergence in 14 non linear iterations
- ✓ Only 3 design variables: 1 collective, 2 cyclics

Optimization Problems – UH60

Unconstrained optimization: Penalty function + retrim



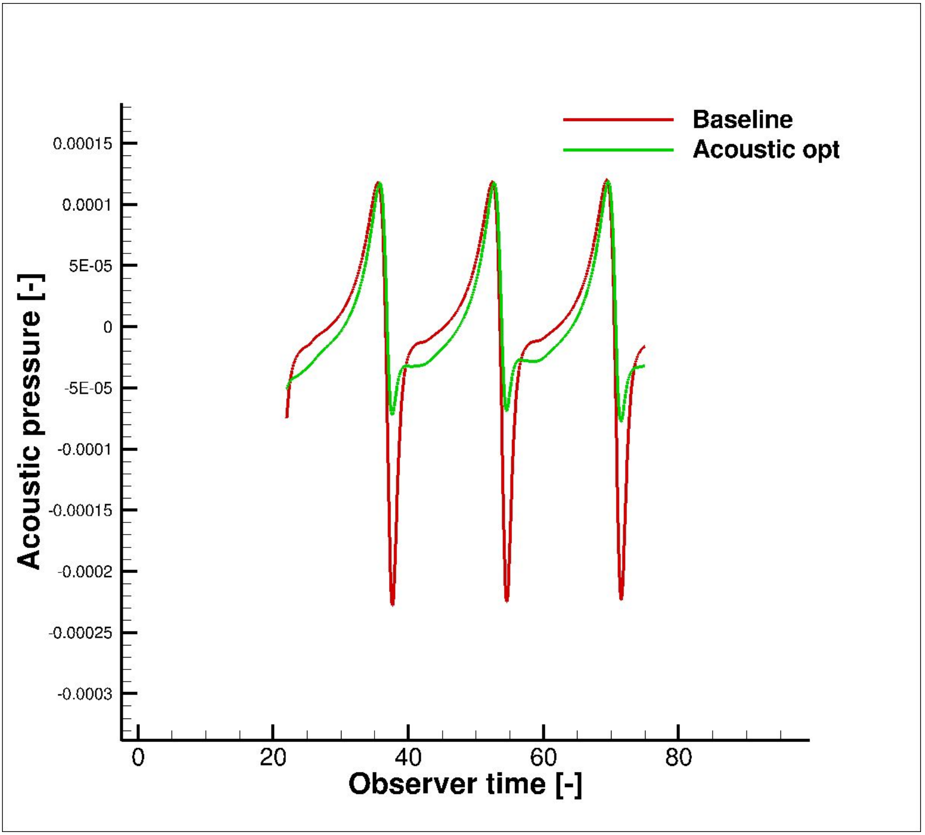




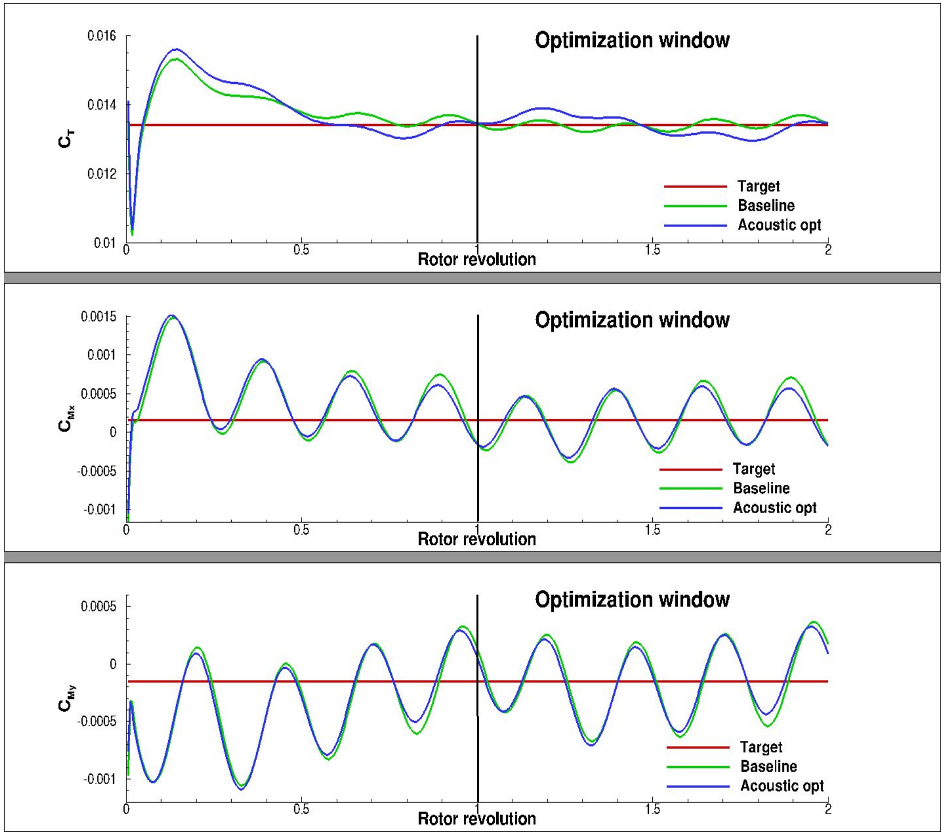
- ✓ Penalty function approach: 1 adjoint
- ✓ 127 design variables
- ✓ Requires final retrim
- ✓ 3.9dB OSPL quieter rotor after retrim

Optimization Problems – UH60

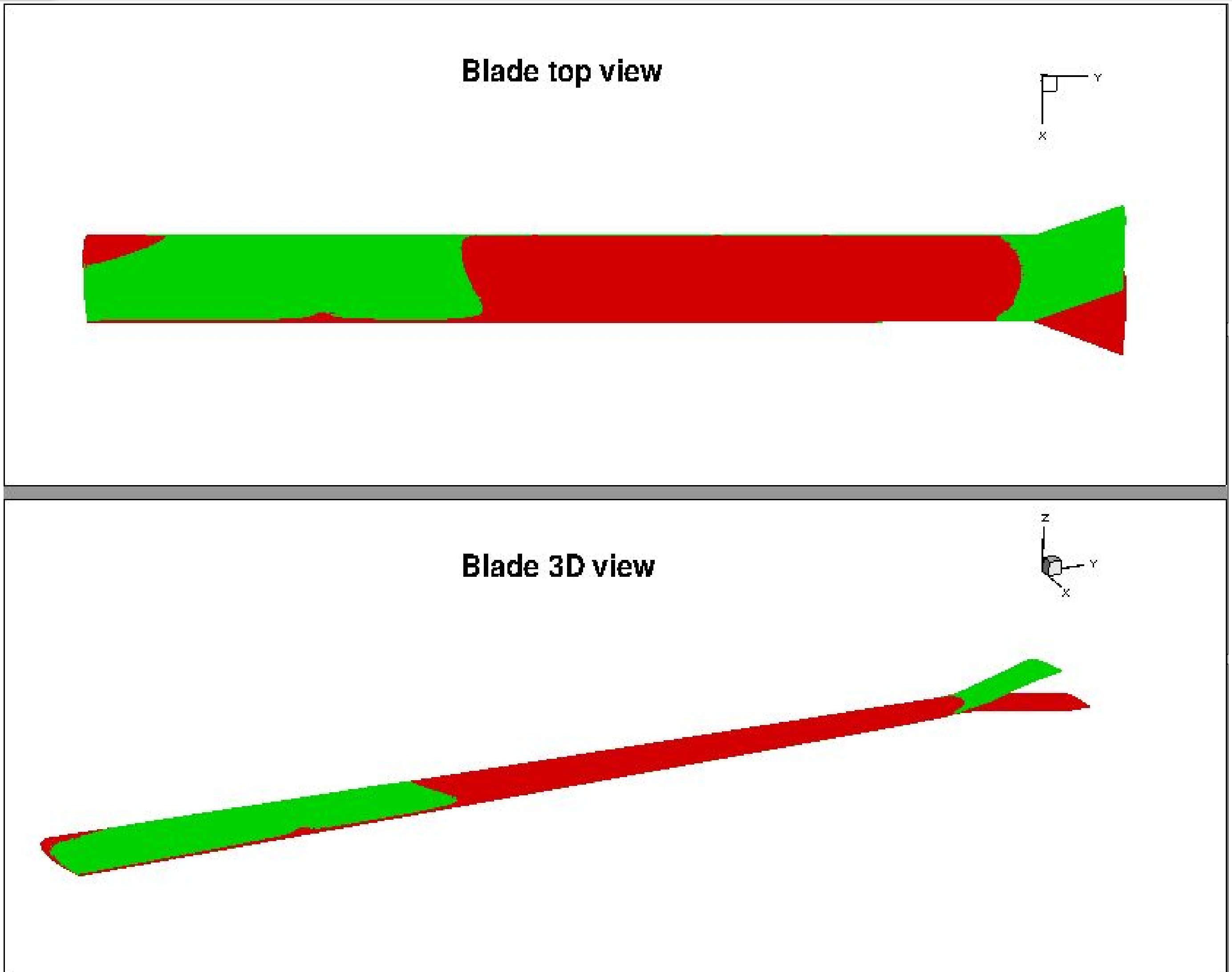
3.9dB OSPL quieter rotor



Final retrim



Optimization Problems – UH60



Conclusions

- Time dependent high-fidelity 3D multidisciplinary suite implemented and verified
  - Aerodynamics, structural mechanics, aeroacoustics
- Time dependent multidisciplinary tangent and adjoint sensitivity verified on HART-II rotor in trimmed forward flight.
- Multidisciplinary adjoint formulation used to **optimize** flexible HART-II and rigid UH60 noise signature.

Original contribution

- Development of a 3D coupled aeroacoustic adjoint for rotorcraft problems
  - Rigid rotor (AIAA Scitech 2016,2017)
  - Flexible rotor (AHS 72nd Annual Forum, May 2016)

- Enable high-fidelity gradient-based aeroacoustic optimization for rotorcraft problem

Future Work

- Finer meshes/multiple rotor revolutions
- Higher fidelity structural model
- Nonlinear flow effects in noise prediction
  - Quadrupole term
- Existence of multiple local minima
  - Hybrid global/local optimization (Gradient enhanced Kriging/Response Surface Models)
- Multipoint design optimization
  - Hover & Forward flight

Future Work

- Linear systems with multiple right hand sides
  - Multiple adjoints / Hessian computation
- Newton's method for optimization problems
- Partial convergence

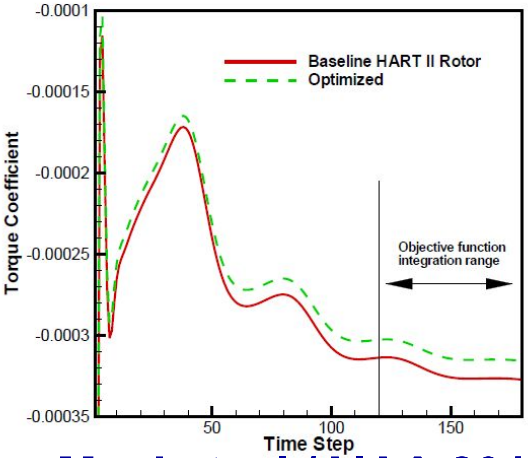
Acknowledgements

- Alfred Gessow Rotorcraft Center of Excellence at University of Maryland
- University of Wyoming ARCC
- NCAR-Wyoming Supercomputer Center
- Dr. K. S. Brentner
- Dr. K. Mani, Dr. A. Mishra

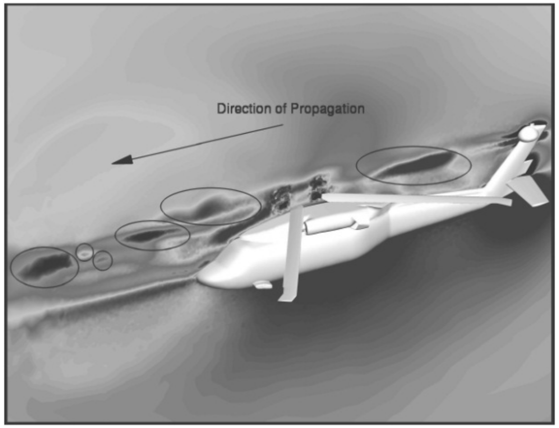
Questions?

Background

- Adjoint optimization: computational cost independent of number of design variables
  - Steady: fixed wing shape optimizations
  - Unsteady: Mavriplis, Mani, Nielsen
  - Aeroacoustic: Rumpfkeil, Economon, Fabiano
  - Helicopter blade design
    - Hover: Mani et. al, Lee et. al
    - Forward flight: Nielsen et al, Choi et al, Alonso et al, Mishra et al.



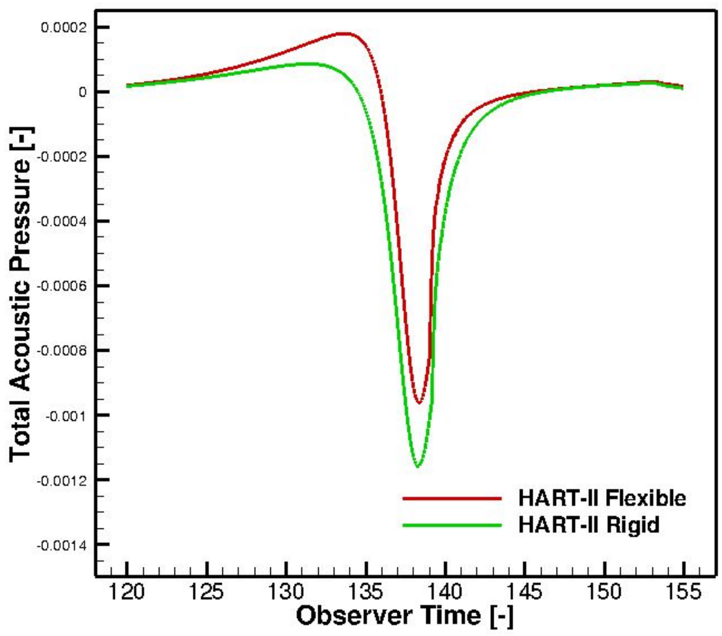
Mani et. al (AIAA 2013)

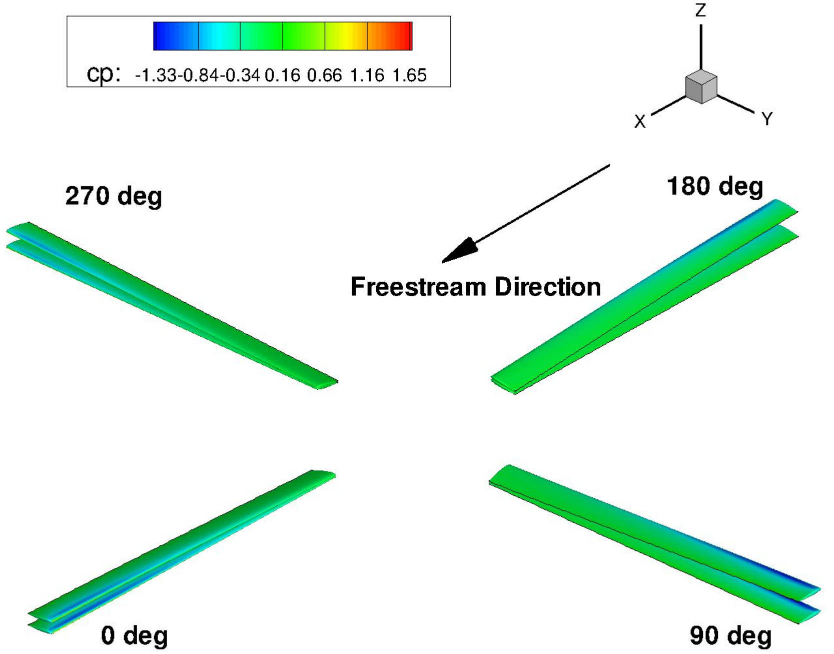


Nielsen et. al (AIAA 2012)

Acoustic problem

Comparison of rigid and flexible rotor noise signature





Acoustic Sensitivity: Tangent

- Acoustic pressure @ far field observer:

$$p'(\mathbf{y},t,\mathbf{D}) = \mathbf{FWH}(\mathbf{U}(\mathbf{D}),\mathbf{x}(\mathbf{D}))$$

- Forward sensitivity of acoustic pressure:

$$\frac{dp(\mathbf{y},t,\mathbf{D})}{dD} = \sum_n \frac{\partial \mathbf{FWH}}{\partial \mathbf{U}^{\mathbf{n}}_{\mathbf{FWH}}}\frac{\partial \mathbf{U}^{\mathbf{n}}_{\mathbf{FWH}}}{\partial D} + \frac{\partial \mathbf{FWH}}{\partial \mathbf{x}^{\mathbf{n}}_{\mathbf{FWH}}}\frac{\partial \mathbf{x}^{\mathbf{n}}_{\mathbf{FWH}}}{\partial D}$$

- Time-integrated acoustic objective function (RMS acoustic pressure):

$$L_{FWH} = p'_{RMS}$$

- Forward sensitivity of time-integrated objective function:

$$\frac{dL_{FWH}}{dD} = \frac{\partial p'_{RMS}}{\partial p'}\frac{\partial p'}{\partial D} = \frac{\partial p'_{RMS}}{\partial p'}\left[\sum_n \frac{\partial FWH}{\partial \mathbf{U}^{\mathbf{n}}_{\mathbf{FWH}}}\frac{\partial \mathbf{U}^{\mathbf{n}}_{\mathbf{FWH}}}{\partial D} + \frac{\partial FWH}{\partial x^n_{FWH}}\frac{\partial x^n_{FWH}}{\partial D}\right]$$

Acoustic Sensitivity: Adjoint

- Acoustic pressure @ far field observer:

$$p'(\mathbf{y}, t, \mathbf{D}) = \mathbf{FWH}(\mathbf{U}(\mathbf{D}), \mathbf{x}(\mathbf{D}))$$

- Adjoint sensitivity of acoustic pressure:

$$\frac{dp(\mathbf{y}, t, \mathbf{D})}{dD}^T = \sum_n \frac{\partial \mathbf{U}^{\mathbf{n}}_{\mathbf{FWH}}}{\partial D}^T \frac{\partial \mathbf{FWH}}{\partial \mathbf{U}^{\mathbf{n}}_{\mathbf{FWH}}}^T + \frac{\partial \mathbf{x}^{\mathbf{n}}_{\mathbf{FWH}}}{\partial D}^T \frac{\partial \mathbf{FWH}}{\partial \mathbf{x}^{\mathbf{n}}_{\mathbf{FWH}}}^T$$

- Time-integrated acoustic objective function (RMS acoustic pressure):

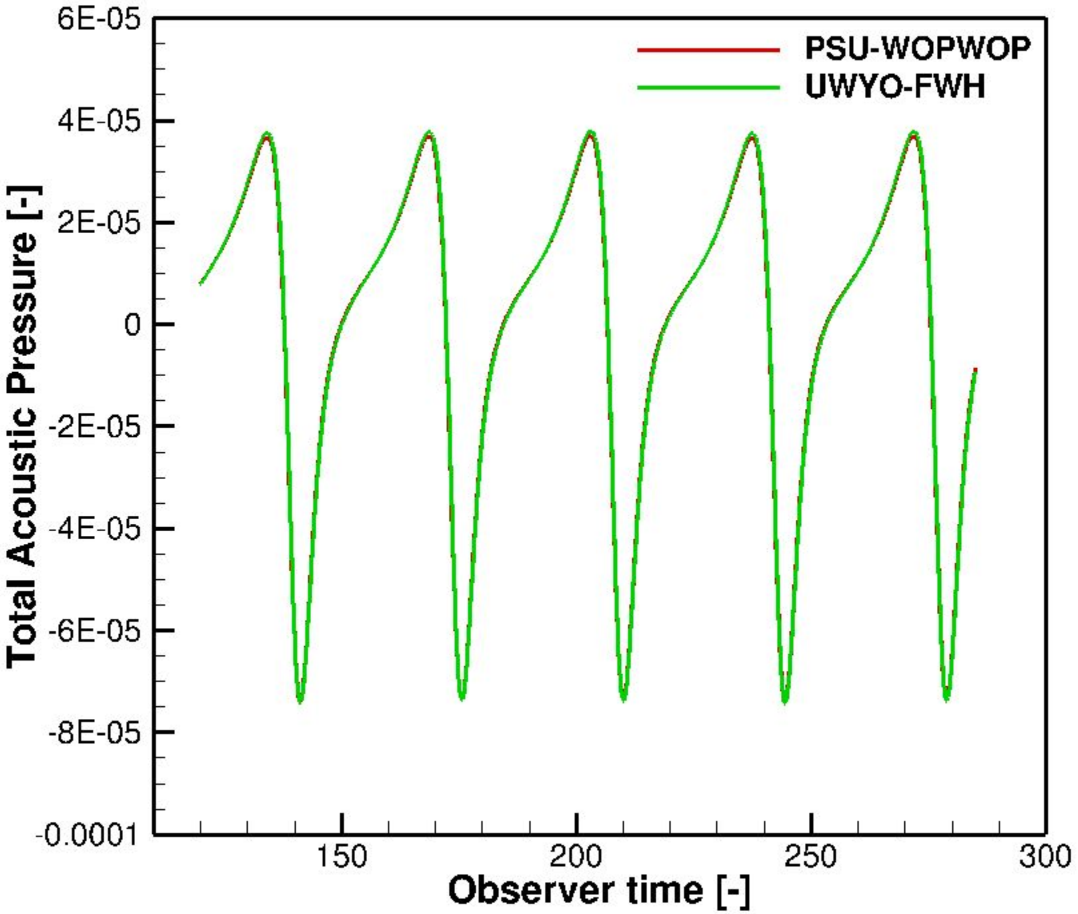
$$L_{FWH} = p'_{RMS}$$

- Adjoint sensitivity of time-integrated objective function:

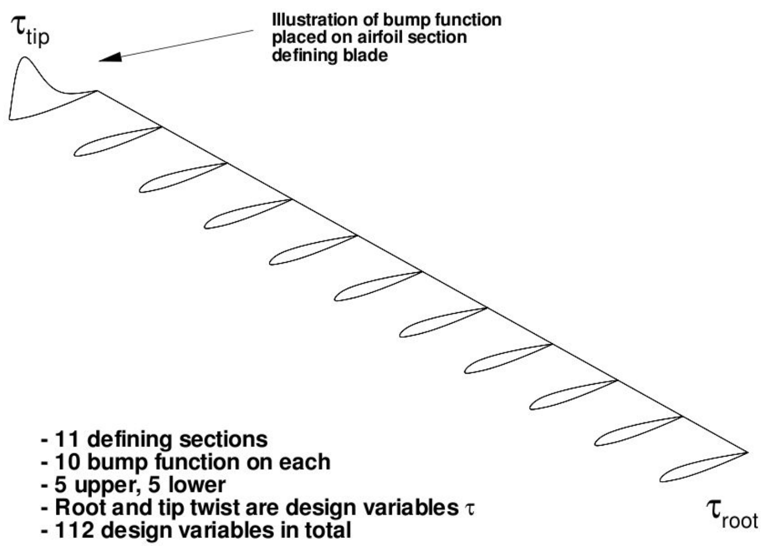
$$\frac{dL_{FWH}}{dD}^T = \frac{\partial p}{\partial D}^T \frac{\partial p'_{RMS}}{\partial p}^T = \sum_n \frac{\partial U^n}{\partial D}^T \frac{\partial L_{FWH}}{\partial U^n}^T + \frac{\partial x^n}{\partial D}^T \frac{\partial L_{FWH}}{\partial x^n}^T$$

Future Work

- Multi
- Extra
- Diffe
  - Wi
  - Qu

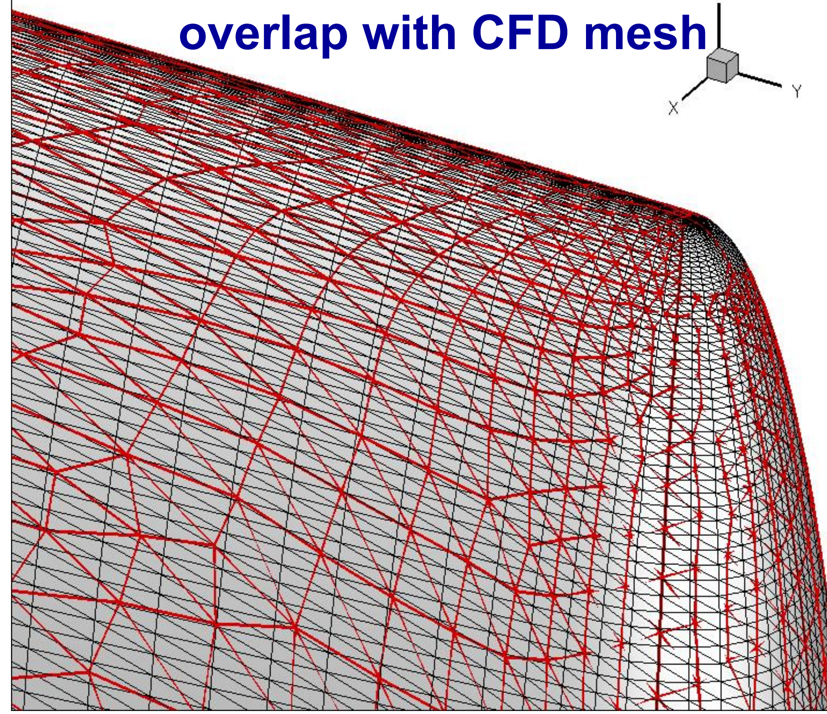


Blade Geometry Parametrization



Hicks-Henne bump functions

Structured blade mesh overlap with CFD mesh



- Master blade shape defined by Hicks-Henne bump functions and twist
  - Defined by high-resolution structured mesh (in black)
  - Shape changes interpolated onto unstructured CFD surface mesh
- 95 design parameters
  - 10 Hicks-Henne bump functions per blade section, 9 blade sections (90)
  - Twist at blade root and tip (2) and 3 pitch parameters