THE DEVELOPMENT OF UNSTRUCTURED GRID METHODS FOR COMPUTATIONAL AERODYNAMICS

Dimitri J. Mavriplis

ICASE
NASA Langley Research Center
Hampton, VA

University of Illinois at Urbana-Champaign
April 29, 2002
MOTIVATION

- Development of Practical Aerodynamic CFD Capability
  - Unstructured Grids for Complex Geometries
  - Algorithmic Research
    * Discretization
    * Solution Techniques
  - Computer Science Research
    * Cache Efficiency
    * (Vector)/Parallel Processing
- Validation on Realistic Aerodynamic Problems
  * NASA Wind Tunnel Data
  * Collaboration with Industry
OVERVIEW

• Unstructured Grid Advantages/Disadvantages

• Discretization

• Solution Procedures
  – Multigrid Methods

• Grid Anisotropy
  – Directional Preconditioning

• Parallelization

• Validation
  – Large Research Cases on Supercomputers
  – Smaller Production Cases on PC Clusters

• Current and Future Topics
OVERVIEW

● (Block) Structured Grids
  – Logically Rectangular
  – Supports Dimensional Splitting Algorithms
  – Banded Matrices
  – Block Structure for Complex Geometries

● Unstructured Grids
  – Lists of Cell Connectivity, Graphs (Edges, Vertices)
  – Alternate Discretization/Solution Strategies
  – Sparse Matrices
  – Complex Geometries, Adaptive Meshing
  – More Efficient Parallelization (homogeneous)
DISCRETIZATION

• Governing Equations: Reynolds Averaged Navier-Stokes
  – Conservation of Mass Momentum and Energy
  – Single Equation Turbulence Model (Spalart-Allmaras)
    * Convection - Diffusion - Production

• Vertex-Based Discretization
  – 2nd order upwind finite-volume scheme
  – 6 variables per grid point
  – Flow equations fully coupled \((5 \times 5)\)
  – Turbulence equation uncoupled
SPATIAL DISCRETIZATION

- **Mixed Element Meshes**
  - Tetrahedra, Prisms, Pyramids, Hexahedra

- **Control Volume Based on Median Duals**
  - Fluxes based on edges
    
    $F_{ik} = f(u_{\text{left}}, u_{\text{right}})$
    
    * $u_{\text{left}} = u_i, u_{\text{right}} = u_k$: 1st order accurate
    
    * $u_{\text{left}} = u_i + \frac{1}{2} \nabla u_i \cdot r_{ik}$
    
    * $u_{\text{right}} = u_k + \frac{1}{2} \nabla u_k \cdot r_{ki}$: 2nd order accurate
    
    * $\nabla u_i$ evaluated as contour integral around CV

- **Single Edge Based Data Structure** represents all element types
SOLUTION OF SPATIALLY DISCRETIZED EQUATIONS

\[ \frac{du}{dt} + R(u) = 0 \]

- Integrate to Steady-State

- Explicit: \[ u^{n+1} = u^n - \Delta t R(u^n) \]
  - Simple
  - Slow Convergence: Local Procedure

- Implicit: \[ \left( \frac{I}{\Delta t} + \frac{\partial R}{\partial u} \right)(u^{n+1} - u^n) = -\Delta t R(u^n) \]
  - Large Memory Requirements

- Matrix-Free Implicit: \[ \frac{\partial R}{\partial u} \Delta u = \frac{R(u) - R(u + \epsilon \Delta u)}{\epsilon} \]
  - Most Effective with Matrix-Based Preconditioner

- Multigrid Methods
CYCLING STRATEGIES

V-Cycle

W-Cycle

$T = \text{Time-Step}$

$R = \text{Restriction}$

$P = \text{Prolongation}$
PARALLEL IMPLEMENTATION

- Intersected Edges Resolved by Ghost Vertices
- Generates Communication between Original and Ghost Vertex
  - Handled using MPI and/or OpenMP
  - Portable, Distributed and Shared Memory Architectures
- Local Reordering within partition for Cache- Locality
PARTITIONING

- Graph Partitioning Must Minimize Number of Cut Edges to Minimize Communication Volume

- Standard Graph Based Partitioners: MeTis, CHACO
  - Require only Weighted Graph Description of Grid
    - Edges, Vertices and Weights (taken as unity)
  - Ideal for Edge Data Structure

- Line Solver Inherently Sequential
  - Partition Around Lines using Weighted Graphs
SAMPLE CALCULATIONS AND VALIDATION

- Subsonic High-Lift Case
  - Geometrically Complex
  - Large Case: 25 million points, 1450 processors
  - Research Environment Demonstration Case

- Transonic Wing Body
  - Smaller Grid Sizes
  - Full Matrix of Mach and $C_L$ conditions
  - Typical of Production runs in design environment
**OBSERVED SPEEDUPS FOR 24.7M PT GRID**

- **Good Multigrid Scalability up to 1450 PEs**
- **Multigrid Scalability Decrease due to Coarse Grid Communication**
  - *(single grid solver not feasible: 100 times slower)*
- **1 hour solution time on 1450 PEs (82 Gflops)**

---

24.7 Million Pt Case
(5 Multigrid Levels)

<table>
<thead>
<tr>
<th>Platform</th>
<th>No. of Procs</th>
<th>Time/Cyc</th>
<th>Gflop/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>T3E-600</td>
<td>512</td>
<td>28.1</td>
<td>22.0</td>
</tr>
<tr>
<td>T3E-1200e</td>
<td>256</td>
<td>38.3</td>
<td>16.1</td>
</tr>
<tr>
<td>T3E-1200e</td>
<td>512</td>
<td>19.7</td>
<td>31.4</td>
</tr>
<tr>
<td>T3E-1200e</td>
<td>1024</td>
<td>10.1</td>
<td>61.0</td>
</tr>
<tr>
<td>T3E-1200e</td>
<td>1450</td>
<td>7.54</td>
<td>82.0</td>
</tr>
</tbody>
</table>
COMPARISON WITH EXPERIMENTAL DATA

- Lift versus Incidence Slightly Over Predicted
- Drag Polar Well Predicted on Fine Grid
- Maximum Lift Point Overpredicted by 1.0 degree
- High Lift Flows among most difficult to predict accurately
  - Geometric Complexity
  - Complex flow physics
  - Extremely fine grids required
TRANSONIC WING BODY TEST CASE

- Test Case for AIAA Drag Prediction Workshop
  - Assess Capability of Modern CFD Methods for Drag Prediction
  - Realistic but Simple Geometry
  - Drag polars, Drag Rise Curves
    * Typical for aircraft design studies

- Grid Resolution Effects

- Rapid Turnaround for Large Number of Cases on Commodity Hardware

- Joint Work with Cessna Aircraft (D. Levy)
• **DLR-F4 Wing-Body Configuration**

• **Supplied Grid, Custom built Grids**

• **Mandatory Cases:**
  - Fixed Point $M=0.75$, $C_L=0.5$, Drag Polar at $M=0.75$

• **Optional Cases**
  - Drag Rise Curves (Drag vs. Mach at constant $C_L$)
CASES RUN

- **BASELINE GRID:** 1.6 million points
  - Full Drag Polars for Mach Numbers: 0.5, 0.6, 0.7, 0.75, 0.76, 0.77, 0.78, 0.8
  - Interpolated Incidence on Polars at Prescribed Lift Value
  - Total: 72 cases

- **MEDIUM GRID:** 3.0 million points
  - Full Drag Polars for Each Mach Number
  - Total: 48 cases

- **FINE GRID:** 13 million points
  - Computed Drag Polar at Mach = 0.75
  - Computed $C_L$=0.5 case at Mach=0.75
  - Total: 7 cases
  - Highest Incidence case not fully converged
SAMPLE SOLUTION ON BASELINE GRID (1.65 M pts)

- Mach = 0.75, $C_L = 0.6$, Re = 3 million
- Baseline Grid (1.65 million points)
• Adequate Boundary Layer Resolution on Baseline Grid

• Force Coefficients Converged in 250 Multigrid Cycles for this case

• All Cases run Minimum of 500 Multigrid Cycles
BASELINE GRID CASES RUN ON ICASE CLUSTER

- Polars for all Mach Numbers: 72 Cases
- 2.5 hours per case on 16 1.7GHz Pentium CPUs
- About 1 week to compute all cases
RESULTS FOR CASE 1: Mach = 0.75, CL=0.5, Re = 3M

<table>
<thead>
<tr>
<th>Case</th>
<th>$C_L$</th>
<th>$\alpha$</th>
<th>$C_D$</th>
<th>$C_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment (ONERA)</td>
<td>0.5000</td>
<td>+.192°</td>
<td>0.02896</td>
<td>-1.301</td>
</tr>
<tr>
<td>Experiment (NLR)</td>
<td>0.5000</td>
<td>+.153°</td>
<td>0.02889</td>
<td>-1.260</td>
</tr>
<tr>
<td>Experiment (DRA)</td>
<td>0.5000</td>
<td>+.179°</td>
<td>0.02793</td>
<td>-1.371</td>
</tr>
<tr>
<td>Grid1(1.6Mpts) (ICASE)</td>
<td>0.5004</td>
<td>-2.241°</td>
<td>0.02921</td>
<td>-1.549</td>
</tr>
<tr>
<td>Grid1(1.6Mpts) (Cessna)</td>
<td>0.4995</td>
<td>-2.248°</td>
<td>0.02899</td>
<td>-1.548</td>
</tr>
<tr>
<td>Grid2(3.0Mpts)</td>
<td>0.5000</td>
<td>-4.17°</td>
<td>0.02857</td>
<td>-1.643</td>
</tr>
<tr>
<td>Grid3(13Mpts)</td>
<td>0.5003</td>
<td>-3.67°</td>
<td>0.02815</td>
<td>-1.657</td>
</tr>
</tbody>
</table>

- **Good Overall Drag Agreement (10 counts)**
- **Notable Incidence Offset**
- **Substantial Overprediction of Lift at Given Incidence**
  - Observed by majority of workshop participants
- **Slope Overpredicted by \( \approx 5\% \)**
- **Unaffected by Grid Resolution**
**Good Drag Prediction Despite** $C_L$ Shift

**Better Agreement at Low** $C_L$ with Increased Grid Resolution
DRAG RISE COMPARISON WITH EXP. DATA (CASE 4)

Notes:
1. Wind tunnel data use prescribed BL trip pattern.
2. CFD data are fully turbulent.
3. On fine grid, even $C_l$ data interpolated from $\alpha$-sweep data using cubic spline.

- Reasonable Overall Comparison for Relatively Coarse Grid
- Increased Discrepancies at Higher Mach Number and Lift
ADDITIONAL DRAG POLARS

- Increased Accuracy for Finer Grid at Lower Lift Values
- Increased Discrepancies at Higher Mach Number and Lift

Note: Wind tunnel data follow prescribed trip pattern; CFD data are fully turbulent.
DRAG UNDERPREDICTION AT HIGH CL/Mach

- Separation Likely Underpredicted at High $C_L$/Mach Conditions
  - Influence of Turbulence Models

- Free Transition in Computations
  - Computationally observed $\approx$ 5% to 7% chord
  - Experimentally Set 15% (upper) and 25% (lower) chord

- Possible Effects due to $C_L$-Incidence Offset
VALIDATION SUMMARY

• Unstructured Grid Methods Comparable and Often Superior to Structured Counterparts
  – Similar Accuracy
  – Reduced Setup Time
  – Good Parallelization Characteristics

• CFD Methods Perform Well at Design Conditions (Attached Flow)

• High Incidence, High Lift More Problematic

• Transition, Turbulence Modeling Important Issues

• Grid Resolution always an Issue
CURRENT AND FUTURE RESEARCH AREAS

- Adaptive Meshing
  - Mixed Element Subdivision
  - Refinement Criteria Pacing Issue
  - Dynamic Load Balancing for Parallel Computing

- Unsteady Flows
  - Implicit Time Solution Procedures
  - Moving Grids, Overlapping Grids
  - Overlapping Grids

- LES and DES Simulations of Separated Flows

- Higher-Order Methods
  - 4th order in Time (Implicit Runge-Kutta)
  - Discontinuous Galerkin, SUPG Methods