Algorithmic Contributions to the CFD2030 Grand Challenge Problems

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University of Wyoming
OVERVIEW

• Introduce Grand Challenge Problems
  – Original notional GCs in CFD2030 report
  – GCs being formulated as part of CFD2030 IC

• Focus on technical challenges to enable GC problems
  – Algorithmic contributions

• Tie back to previous work of A. Jameson
  – Grand Challenges
  – Capabilities enabled by algorithmic contributions of A. Jameson
CFD Vision 2030 Study

- **Elements of the study effort:**
  - Define and develop **CFD requirements**
  - Identify the most critical **gaps and impediments**
  - Create the **vision**
  - Develop a long-term, actionable **research plan** and detailed **technology development roadmap**

- **Executed user survey and technical workshop**

- **Comprehensive final report** – **NASA CR 2014-218178**
  - Provides a detailed CFD vision and technology outlook, including assessment of High Performance Computing (HPC)
  - **Guides future CFD technology development** at NASA and within the broader CFD community
  - Being used as an **advocacy document** to drive the implementation of the CFD vision
CFD Vision 2030 Roadmap

HPC
- CFD on Massively Parallel Systems
  - PETASCALE
- CFD on Revolutionary Systems (Quantum, Bio, etc.)
  - EXASCALE

Physical Modeling
- RANS
- Hybrid RANS/LES
- LES
- Combustion
  - Integrated transition prediction
  - Chemical kinetics calculation speedup

Algorithms
- Convergence/Robustness
- Uncertainty Quantification (UQ)
  - Characterization of UQ in aerospace
  - Reliable error estimates in CFD codes
- Grid convergence for a complete configuration
- Multi-regime turbulence-chemistry interaction model
- Production scalable entropy-stable solvers

Geometry and Grid Generation
- Fixed Grid
  - Tighter CAD coupling
- Adaptive Grid
  - Production AMR in CFD codes
- Integrated Databases
  - Simplified data representation

Knowledge Extraction
- Visualization
  - On demand analysis/visualization of a 10B point unsteady CFD simulation
- MDAO
  - MDAO simulation of an entire aircraft (e.g., aero-acoustics)

MDAO
- Define standard for coupling to other disciplines
- High-fidelity coupling techniques
- Robust CFD for complex MDAs
- MDAO simulation of an entire aircraft (e.g., aero-acoustics)
- UQ-Enabled MDAO

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Grand Challenge Problems

- Highlight critical **step changes** needed in engineering design capability
- May **not be routinely achievable** by 2030
- Represent key elements of **major NASA missions**

1. Large Eddy Simulation (LES) of a powered aircraft configuration across the full flight envelope
2. Off-design turbofan engine transient simulation
3. Multi-Disciplinary Analysis and Optimization (MDAO) of a highly-flexible advanced aircraft configuration
4. Probabilistic analysis of a powered space access configuration


Credit: The Boeing Company
Proposed GC Problems under CFD2030 IC

- High Lift Wind up Turn
- High-Fidelity CFD Based Compressor Performance Map
- CFD-in-the-Loop Monte Carlo Flight Simulation for Space Vehicle Design
- Hypersonics Grand Challenge

• Special Session at Aviation 2020
Motivation

• Consideration of Specific GC problem
  – Based on a value proposition: What if?
  – Identify technical barriers
    • Algorithmic contributions
  – Identify logistical barriers
    • e.g. Computational resources, software engineering
  – Focus resources
  – Promote collaboration towards shared objective
    • CFD technology
    • Meshing technology
    • Disciplinary coupling
    • Uncertainty Quantification
    • Visualization/Knowledge extraction
Grand Challenge of the 1980’s:
Full Aircraft CFD Simulation

• Wing or wing body configurations SOA
  – Single or multi-block structured meshes
• Extensions to wing-pylon-nacelle difficult
• Extensions to 3D high-lift configurations considered intractable
• Required a rethinking of current approaches
  – Unstructured meshes
1986-87 Jameson Airplane Papers

- Unstructured tetrahedral mesh
  - 35,370 points, 181,959 tetrahedra
  - Mesh generation: 15 minutes
    - No mention of geometry issues
  - Flow solver: 1 hour on 1 processor of CRAY-XMP
    - Vectorized, later parallelized for CRAY-XMP/YMP
1986-87 Jameson Airplane Papers

• Unstructured tetrahedral mesh
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Jameson Airplane

- Essentially a self-created GC problem
- Largely enabled through algorithmic advances
- Required collaboration and advances on various fronts
  - Mesh generation
    - Delaunay triangulation/Surface recovery
  - Discretization
    - JST scheme on tetrahedral elements/Edge based data structure
  - Parallel computing
    - Cray multitasking
Current Day GC: 
High Lift Wind Up Turn

• Aircraft Maneuver at Edge of Flight Envelope
  – Demonstration for design
  – Implications for certification by analysis (CbA)

• Characteristics:
  – Multidisciplinary
    • Aerodynamics, structures, controls
  – Flow physics
    • Stall, buffet, smooth body separation

• Break down into series of challenge problems of increasing difficulty
Advancing High Lift Aerodynamic Prediction
Series of Challenge Problems

Sub-Challenge Problem #1
1-3 years

- CRM-HL Ecosystem
  - CRM-HL + EMPAS
    - Challenge Problem #1 + S&C (control surfaces/trim)
    - Gear effects
    - Cross-flow
    - Power effects
    - Ice prediction

- CRM-HL
  - Landing/TO configuration
  - Up to flight Re
  - Flow physics (separation, vortical flow)
  - Static aeroelastics
  - Ice effects

Sub-Challenge Problem #2
3-6 years

- CRM-HL + EMPAS
  - Challenge Problem #2
  - S&C (control surfaces/trim)
  - Gear effects
  - Cross-flow
  - Power effects
  - Ice prediction

Sub-Challenge Problem #3
6-10+ years

Grand Challenge Problem
15+ years

CRITICAL MANEUVER

LOW-SPEED WIND-UP TURN (or similar)

Sub-Scale Generic Flight Vehicle
- Flight Re
- Flight geometry
- Dynamic, maneuvering flight
- Dynamic structural response
- CFD-generated data populates flight simulation database*

Full-Scale Generic Flight Vehicle
- Flight Re
- Flight geometry
- Dynamic, maneuvering flight
- Dynamic structural response
- Environmental effects
- Full engine simulation
- CFD-based flight simulation**

* Accuracy determined by proof-of-match between flight simulation and flight data
** Flight test used to verify flight simulation

EMPAS = Electric Motor Powered AeroEngine Simulator

NASA AirSTAR
Current Day GC:
High Lift Wind Up Turn

• Logistical Technical Challenges
  – Software coupling of all relevant disciplines
  – Parallel efficiency, emerging hardware trends
  – Software engineering and maintainability
  – Traceable and reproducible (CbA)

• Algorithmic Technical Challenges
  – Very high resolution required
    • Highly detailed water-tight CAD with automatic defeaturing
    • Multi-Billion cell grids/Curved Elements
    • High-order discretizations
    • Efficient implicit solvers
  – Relative geometry motion
    • Dynamic AMR meshes
  – Ability to predict relevant flow physics
    • Scale resolving methods with suitable subgrid scale models
  – Uncertainty Quantification (UQ)
  – In-situ visualization/Knowledge extraction/ROMs
Substantial Advances in Digital Flight

CREATE-AV

- Leveraged dynamic overset, AMR, higher order, multidisciplinary
- Digital fight for rotorcraft even more challenging
GMGW Meshing Challenge

Case 1: Exascale Meshing of the HL-CRM

Goal
Attempt to generate an Order 10.5 (aka "2019 Hero" resolution, 31 billion cell) mesh for the HL-CRM rev. 2 geometry model.

Case 1 is designed to break our tools and processes in order to learn what needs to be fixed before the year 2030 when Order 10.5 will be Medium resolution, not Hero resolution.

Participants are asked to generate the largest mesh they can up to Order 10.5 and use the Participant Questionnaire (see below) to describe where they encountered problems.

Geometry Model
Download the HL-CRM rev. 2 geometry model from the workshop ftp site.

Right click on the file link and use Save link as.
- NX
- Parasolid
- STEP
- IGES

If you must or prefer to use command line ftp, follow these instructions.
- ftp files.gmgworkshop.com

- Billion cell meshes
- Curved element meshes
- CFD2030 driven
GMGW Meshing Challenge

Mesh Order Description

- Order = \( \log_{10} \) (Mesh Size)
- E.g., 3.16 billion cell mesh = \( \log_{10}(3.16 \times 10^{10}) = \) order 9.5
- This year’s Hero mesh (Order 10.5, 31.6 billion cells) will be considered a medium mesh by 2030
- Participants successfully generated meshes in the order 9.2 (1.7 billion cell) to order 9.9 (7.9 billion cell) range

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<thead>
<tr>
<th>Order</th>
<th>Description</th>
<th>Num. Cells (billions)</th>
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<tbody>
<tr>
<td>8.0</td>
<td>Coarse</td>
<td>0.100</td>
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<tr>
<td>8.5</td>
<td>Medium</td>
<td>0.316</td>
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<tr>
<td>9.0</td>
<td>Fine</td>
<td>1.000</td>
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<tr>
<td>9.5</td>
<td>Extra Fine</td>
<td>3.160</td>
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<tr>
<td>10.0</td>
<td>Super Fine</td>
<td>10.000</td>
</tr>
<tr>
<td>10.5</td>
<td>Hero</td>
<td>31.600</td>
</tr>
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GMGW-2, San Diego CA, January 2019
Mesh Order Description

- Order = $\log_{10}$ (Mesh Size)
- E.g., 3.16 billion cell mesh = $\log_{10}(3.16\times10^{10}) = 10.5$
- This year’s Her will be consid...
- Participants sub order 9.2 (1.7) range

Summary

- Machines had more than enough resources for generating/manipulating geometry and surface meshes.
- Volume Mesh Generation/Export was a different story
  - Largest mesh to be successfully generated and exported was Order 9.9
  - Participant C successfully generated Order 10.1 mesh but export failed due to lack of RAM
- No participant was able to achieve an order 10.5 (hero) mesh
  - Lack of RAM (4 participants)
  - Algorithms lack necessary integer support (1 participant)
  - Export (2 participants)
  - Bug in ParMetis partitioning algorithm (1 participant)
Significant Advances in Adaptive Mesh Refinement (AMR) Capabilities

- Enabled though more rigorous algorithms
  - Error estimates (possibly using adjoint methods)
  - Mappings based on continuous spaces
  - Efficient local mesh refinement/improvement operations
Computational Rates for High Order DG Discretizations

- Less computationally intensive than general formulation
- Overall cost much lower per degree of freedom
  - Cost per d.o.f decreases or flat with larger $p$
  - Faster than finite-difference
Tensor Product DG

• Abandon flexibility of modal bases for arbitrary element types

\[ \psi(\xi, \eta, \zeta) = a + b\xi + c\eta + d\zeta + e\xi^2 + f\xi\eta + ... \]

• Tensor product bases:
  – Best suited for hexahedral elements

\[ \psi_{ijk}(\xi, \eta, \zeta) = l_i(\xi)l_j(\eta)l_k(\zeta) \]

• \(l_i, l_j, l_k\) = 1-D Legendre polynomials:
  – values at quadrature points of integration become solution values
  – Removes requirement of reconstructing solution at quadrature points
  – All integrals reduce to dimension-by-dimension 1-D summations
Tensor Product DG

- Abandon flexibility of modal bases for arbitrary element types
  - Cost: $O(N^2)$ or $(p+1)^6$

- Tensor product bases:
  - Cost: $O(N^{4/3})$ or $(p+1)^4$
    - $N = \text{dof per cell} = (p+1)^3$ in 3D
    - $p = \text{basis polynomial degree}$
    - Order of accuracy = $p+1$

- Shown to be equivalent in cost to finite differences on cartesian mesh of same order (for residual evaluation)
- Increasing $p$ at fixed number of d.o.fs
  - Coarser meshes at higher $p$
  - Accuracy increases
  - Simulation cost decreases (per time step)
S-76 Rotor using High-order DG in Off-body Region with AMR

- $p_{max} = 3$  19 secs per time step
- $P_{max} = 7$  12 secs per time step

explicit time step on 5400 cores
Implicit Methods for High p-Order

• Tensor product much more efficient for residual evaluation of high p-order discretizations
  – Ideal for explicit methods
• Implicit methods may require forming/inverting Jacobians
  – Prohibitive cost since element matrix is dense
    • Scales as : $O(N^2)$ or $(p+1)^6$ (at a minimum)
• Efficient implicit solver must rely on tensor product operations to be competitive at high p-order
  – Tensor-product preconditioning (Murman et al.: EDDY code)
  – Pseudo-time stepping (Vincent et al.: PyFR code)
  – P-multigrid
    • Use pseudo-time stepping on sequence of meshes or p-levels
Use explicit pseudo-time stepping on each level
  – Preserves benefits of tensor product formulation

Geometric multigrid at p=0 omitted
  – Frequencies greater than cell size not damped
  – Potentially large cell sizes at p=8
p-MG Solution of Ringleb Flow at $p=5$

- Cost of p-MG cycle at higher $p$
- Largest benefits on finer grid
p-MG Solution of Ringleb Flow at \( p=8 \)

- Cost of p-MG cycle at higher \( p \)
- Largest benefits on finer grid
Adjoint Methods

- Pioneered in aerodynamics by A. Jameson
  - Enables sensitivity computation independent of number of design variables
  - Enabled cost-effective high-fidelity MDAO

- Enables AMR based on engineering objectives rather than local error

- Used to compute sensitivities for UQ (CbA)

- Build better response surface models or ROMS
Aerodynamic Data Base Fill In

- Lift = \( f(\text{Mach}, \alpha) \)
  - Exact: 144 steady-state solutions
  - Kriging model: 10 solutions
    - Function, Function + Gradient (Adjoint)
Aerodynamic Data Base Fill In

- Drag = f(Mach, alpha)
  - Exact: 144 steady-state solutions
  - Kriging model: 10 solutions
    - Function, Function + Gradient (Adjoint)
Importance of Algorithmic Advances

• Increased simulation capabilities due to:
  – More capable hardware (Moore’s Law)
  – Advanced algorithms

• Algorithmic advances are asymptotic
  – Provide increasing benefits for larger problems
    • $O(N)$ vs $O(N^2)$ when $N=10^{12}$
A. Jameson Contributions
Past and Present

- Full aircraft using unstructured meshes
- Discretizations
  - JST schemes, early extension to unstructured meshes
- Multigrid methods
- Adjoint methods
- High-order methods (DG, FR)
Some Outstanding Algorithmic Challenges

• Predicting smooth body separation
• Reliable transition prediction
• Adjoint techniques for chaotic problems
• High-order nonlinearly stable schemes
  – Explicit and implicit
• Uncertainty quantification
Continued Advocacy for Algorithmic Advances

The four components of the recommended program are:

1. Increased access for the scientific and engineering research community through high bandwidth networks to adequate and regularly updated supercomputing facilities and experimental computers;

2. Increased research in computational mathematics, software, and algorithms necessary to the effective and efficient use of supercomputer systems;

3. Training of personnel in scientific and engineering computing; and

4. Research and development basic to the design and implementation of new supercomputer systems of substantially increased capability and capacity, beyond that likely to arise from commercial requirements alone.
# List of Ph.D. Students' Theses

<table>
<thead>
<tr>
<th>Ph.D. Student</th>
<th>University</th>
<th>Year</th>
<th>Dissertation Title</th>
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<tbody>
<tr>
<td>I-Chung Chang</td>
<td>NYU</td>
<td>1981</td>
<td>Unsteady transonic flow past airfoils in rigid body motion</td>
</tr>
<tr>
<td>Bryan McCarroll</td>
<td>NYU</td>
<td>1982</td>
<td>Theory, computation and application of exponential splines</td>
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<tr>
<td>Richard Pos</td>
<td>Princeton</td>
<td>1983</td>
<td>Transonic flow calculations using triangular finite elements</td>
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<tr>
<td>John Fix</td>
<td>Princeton</td>
<td>1983</td>
<td>On the design of airfoils in transonic flow using the Euler equations</td>
</tr>
<tr>
<td>Seokkeun Yoon</td>
<td>Princeton</td>
<td>1983</td>
<td>Numerical solution of the Euler equations by implicit schemes with multiple grids</td>
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<tr>
<td>Craig Swett</td>
<td>Princeton</td>
<td>1983</td>
<td>A spectral method for the solution of transonic potential flow about an arbitrary two-dimensional airfoil</td>
</tr>
<tr>
<td>Dimitri Marovits</td>
<td>Princeton</td>
<td>1987</td>
<td>Solution of the two dimensional Euler equations on unstructured triangular meshes</td>
</tr>
<tr>
<td>Venkat Venkatakrishnan</td>
<td>Princeton</td>
<td>1987</td>
<td>Computation of unsteady transonic flows over moving airfoils</td>
</tr>
<tr>
<td>Luigi Martuscelli</td>
<td>Princeton</td>
<td>1987</td>
<td>Calculations of viscous flows with a multigrid method</td>
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<tr>
<td>Mohan Jayaram</td>
<td>Princeton</td>
<td>1990</td>
<td>Solution of the three-dimensional Navier-Stokes equations for transonic flow using a multigrid method</td>
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<tr>
<td>Takashi Sakata</td>
<td>Princeton</td>
<td>1990</td>
<td>Solution of the Euler equations in multiphase flow fields using the overlapping-mesh method</td>
</tr>
<tr>
<td>Mark Stewart</td>
<td>Princeton</td>
<td>1990</td>
<td>Non-overlapping composite meshes for multi-element airfoils</td>
</tr>
<tr>
<td>Feng Liu</td>
<td>Princeton</td>
<td>1991</td>
<td>Numerical calculation of turbomachinery cascades flows</td>
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<tr>
<td>Todd Nity</td>
<td>Princeton</td>
<td>1993</td>
<td>Development of a Delaunay-based advection scheme with applications to complex three-dimensional rotational flows</td>
</tr>
<tr>
<td>James Farmar</td>
<td>Princeton</td>
<td>1993</td>
<td>A finite volume multigrid solution to the three dimensional nonlinear ship wave problem</td>
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<tr>
<td>James Reuther</td>
<td>UC Davis</td>
<td>1996</td>
<td>Aerodynamic shape optimization using control theory</td>
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<tr>
<td>Juan Alonso</td>
<td>Princeton</td>
<td>1997</td>
<td>Parallel computation of unsteady and aerodynamic flows using an implicit multigrid-driven algorithm</td>
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<tr>
<td>Andrew Belov</td>
<td>Princeton</td>
<td>1997</td>
<td>An implicit multigrid-driven algorithm for unsteady incompressible flow calculations on parallel computers</td>
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<tr>
<td>Changxin Kim</td>
<td>Princeton</td>
<td>1997</td>
<td>Robust and accurate numerical methods for high speed unsteady flows</td>
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<tr>
<td>Scott Saffier</td>
<td>Princeton</td>
<td>1997</td>
<td>Parallel computation of super sonic reactive flows with detailed chemistry including viscous and species diffusion effects</td>
</tr>
<tr>
<td>Bing-Hong Liu</td>
<td>Princeton</td>
<td>1999</td>
<td>Calculation of nonlinear free surface wave with a fully implicit multigrid method</td>
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<tr>
<td>Paul Liu</td>
<td>Princeton</td>
<td>2001</td>
<td>Two-dimensional implicit time dependent calculations for incompressible flows on unstructured meshes</td>
</tr>
<tr>
<td>Yaqin Fei Xuan</td>
<td>Stanford</td>
<td>2002</td>
<td>Shocks capturing schemes with gas-kinetic methods</td>
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<tr>
<td>Srinan Shankaran</td>
<td>Stanford</td>
<td>2003</td>
<td>Numerical analysis and design of unsteady sails</td>
</tr>
<tr>
<td>Siya Narendra</td>
<td>Stanford</td>
<td>2003</td>
<td>The discrete adjoint approach to aerodynamic shape optimization</td>
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<tr>
<td>Matthew McKibben</td>
<td>Stanford</td>
<td>2003</td>
<td>The applications of non-linear hyperbolic domain methods to the Euler and Navier-Stokes equations</td>
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<tr>
<td>John Hua</td>
<td>Stanford</td>
<td>2003</td>
<td>An implicit-explicit flow solver for compressible flows</td>
</tr>
<tr>
<td>Kasim Leventakis</td>
<td>Stanford</td>
<td>2005</td>
<td>Wing platform optimization using an adjoint method</td>
</tr>
<tr>
<td>Balaji Shrivastava</td>
<td>Stanford</td>
<td>2006</td>
<td>The BGD and LKS schemes for computing Euler and Navier-Stokes flows</td>
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<tr>
<td>George Yang</td>
<td>Stanford</td>
<td>2007</td>
<td>A kinetic scheme for the Navier-Stokes equations and high-order methods for hyperbolic conservation laws</td>
</tr>
<tr>
<td>Arvind Gopinath</td>
<td>Stanford</td>
<td>2007</td>
<td>Efficient Fourier-based algorithms for the time-periodic unsteady problems</td>
</tr>
<tr>
<td>Kartik Palaniappan</td>
<td>Stanford</td>
<td>2007</td>
<td>Algorithms for automatic feedback control of aerodynamic flows</td>
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<tr>
<td>Navin Borujustm</td>
<td>Stanford</td>
<td>2008</td>
<td>Time spectral method for rotorcraft flow with vorticity confinement</td>
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<tr>
<td>Aaron Kramm</td>
<td>Stanford</td>
<td>2009</td>
<td>Mediation for control of dynamic systems</td>
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<tr>
<td>Jan-Dar Lee</td>
<td>Stanford</td>
<td>2009</td>
<td>NLF wing design by adjoint method and automatic transition prediction</td>
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<tr>
<td>Rui Hu</td>
<td>Stanford</td>
<td>2009</td>
<td>Supersonic blimp design via adjoint method</td>
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<tr>
<td>Sachin Premavutnan</td>
<td>Stanford</td>
<td>2010</td>
<td>Towards an efficient and robust high order accurate flow solver for viscous compressible flow</td>
</tr>
<tr>
<td>Samir Karmarkar</td>
<td>Stanford</td>
<td>2011</td>
<td>Mesh adaptive strategies for two-dimensional flows</td>
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<tr>
<td>Kwang Yu Chiu</td>
<td>Stanford</td>
<td>2011</td>
<td>A conservative meshless framework for conservation laws with applications</td>
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<tr>
<td>Yves Allama</td>
<td>Stanford</td>
<td>2012</td>
<td>Energy conserving numerical methods for the computation of complex</td>
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<tr>
<td>Patrick Castonguay</td>
<td>Stanford</td>
<td>2012</td>
<td>High-order energy stable flux reconstruction schemes for fluid flow simulations on unstructured grids</td>
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<tr>
<td>Kui Ou</td>
<td>Stanford</td>
<td>2012</td>
<td>High-order methods for unsteady flows on unstructured grids</td>
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<tr>
<td>Andre Chai</td>
<td>Stanford</td>
<td>2012</td>
<td>Control and suppression of laminar vortex shedding off two-dimensional bluff bodies</td>
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<tr>
<td>Yi Li</td>
<td>Stanford</td>
<td>2013</td>
<td>Automatic mesh adaptation using the continuous adjoint approach and the spectral difference method</td>
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<tr>
<td>Matthew Cullison</td>
<td>Stanford</td>
<td>2013</td>
<td>High fidelity optimization of flapping airfoils and wings</td>
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<tr>
<td>David Williams</td>
<td>Stanford</td>
<td>2013</td>
<td>An overview of time-spectral method for relative motion</td>
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<tr>
<td>Joshua Legal</td>
<td>Stanford</td>
<td>2014</td>
<td>Shape optimization in adaptive search spaces</td>
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<tr>
<td>Georgia Anderson</td>
<td>Stanford</td>
<td>2015</td>
<td>Towards industry-ready high-order flow solvers: increasing robustness and usability</td>
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<tr>
<td>Manuel Lopez-Morales</td>
<td>Stanford</td>
<td>2016</td>
<td>Analysis and design of optimal viscous finite element schemes</td>
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<tr>
<td>Arindam Chakraborty</td>
<td>Stanford</td>
<td>2016</td>
<td>An analysis of stability of the flux reconstruction formulation with applications to shock capturing</td>
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<tr>
<td>Abhishek Shankar</td>
<td>Stanford</td>
<td>2016</td>
<td>On the development of the direct flux reconstruction scheme for high-order flow simulation schemes</td>
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<tr>
<td>Joshua Romano</td>
<td>Stanford</td>
<td>2017</td>
<td>Numerical analysis and implicit time stepping for high-order, fluid flow simulations on GPU architectures</td>
</tr>
<tr>
<td>Terry Watkins</td>
<td>Stanford</td>
<td>2017</td>
<td>Towards industry-ready high-order overset methods on modern hardware</td>
</tr>
<tr>
<td>Jacob Crabill</td>
<td>Stanford</td>
<td>2018</td>
<td>Aerodynamic design of active flow control systems aimed towards drag reduction in heavy vehicles</td>
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